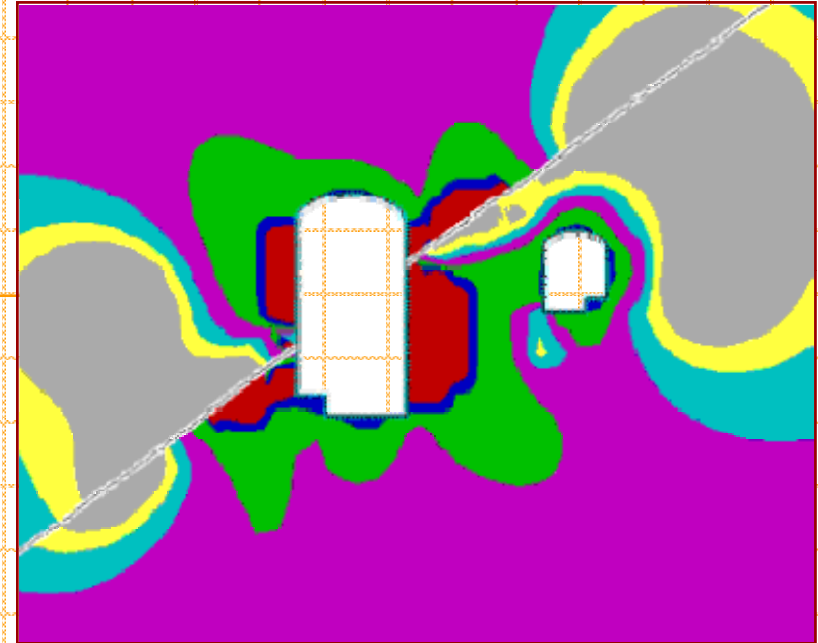# Rock Engineering Practice & Design

Lecture 8:

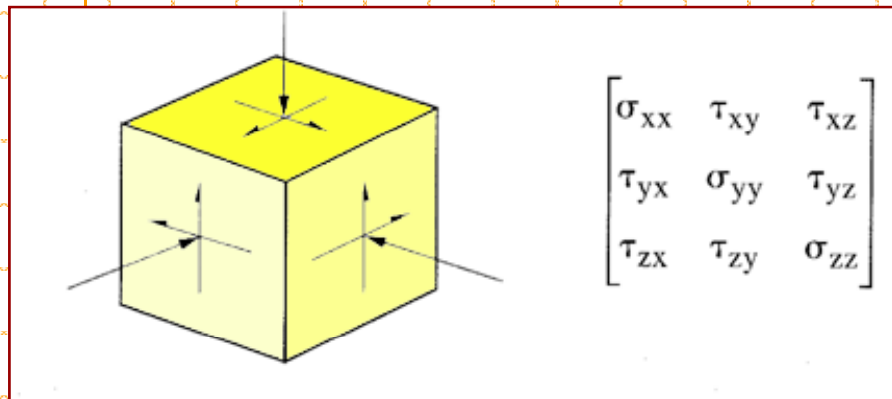## Stress Analysis around Underground Openings

Author's Note:

The lecture slides provided here are taken from the course "Geotechnical Engineering Practice", which is part of the 4th year Geological Engineering program at the University of British Columbia (Vancouver, Canada). The course covers rock engineering and geotechnical design methodologies, building on those already taken by the students covering Introductory Rock Mechanics and Advanced Rock Mechanics.

Although the slides have been modified in part to add context, they of course are missing the detailed narrative that accompanies any lecture. It is also recognized that these lectures summarize, reproduce and build on the work of others for which gratitude is extended. Where possible, efforts have been made to acknowledge the various sources, with a list of references being provided at the end of each lecture.
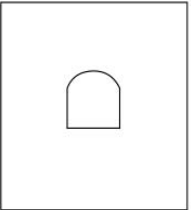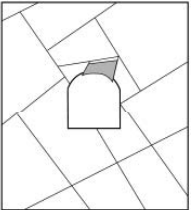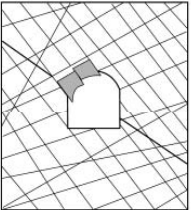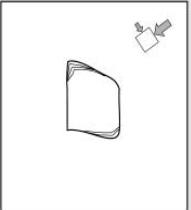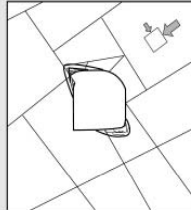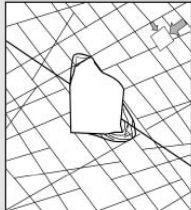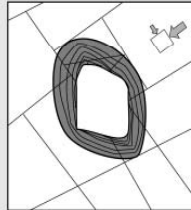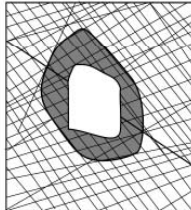
Errors, omissions, comments, etc., can be forwarded to the author at: erik@eos.ubc.ca

Stress-Controlled Instability Mechanisms

Structurally-controlled instabilities are generally driven by a unidirectional body force, i.e. gravity. Stress-controlled instabilities, however, are not activated by a single force, but by a tensor with six independent components. Hence, the manifestations of stress-controlled instability are more variable and complex than those of structurally-controlled failures.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

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	Massive $(RMR > 75)$	Moderately Fractured $(50 > RMR < 75)$	Highly Fractured $(RMR < 50)$	
Low In-Situ Stress $(\sigma_1 / \sigma_c < 0.15)$	Linear elastic response.	Falling or sliding of blocks and wedges.	Unravelling of blocks from the excavation surface.	Low Mining-Induced Stress $\sigma_{max}/\sigma_c < 0.4\pm0.1$
Intermediate In-Situ Stress $(0.15 > \sigma_1 / \sigma_c < 0.4)$	Brittle failure adjacent to excavation boundary.	Localized brittle failure of intact rock and movement of blocks.	Localized brittle failure of intact rock and unravelling along discontinuities.	Intermediate Induced Stress $0.4\pm0.1 < \sigma_{max}/\sigma_c < 1.15\pm0.1$
High In-Situ Stress $(\sigma_1 / \sigma_c > 0.4)$	Failure Zone. Brittle failure around the excavation.	Brittle failure of intact rock around the excavation and movement of blocks.	Squeezing and swelling rocks. Elastic/plastic continuum.	High Mining-Induced Stress $\sigma_{max}/\sigma_c > 1.15\pm0.1$

Kaiser *et al.* (2000)

Stress-Controlled Instability Mechanisms

Although the fundamental complexity of the nature of stress has to be fully considered in the design of an underground excavation, the problem can be initially simplified through the assumptions of continuous, homogeneous, isotropic, linear elastic behaviour (CHILE).

CHILE: Continuous, Homogeneous, Isotropic, Linear Elastic

DIANE: Discontinuous, Inhomogeneous, Anisotropic, Non-Elastic

The engineering question is whether a solution based on the CHILE assumption are of any assistance in design. In fact though, many CHILE-based solutions have been used successfully, especially in those excavations at depth where high stresses have closed the fractures and the rock mass is relatively homogeneous and isotropic. However, in near-surface excavations, where the rock stresses are lower, the fractures more frequent, and the rock mass more disturbed and weathered, there is more concern about the validity of the CHILE model.

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Stress-Controlled Instability Mechanisms

A stress analysis begins with a knowledge of the **magnitudes** and **directions** of the *in situ* stresses in the region of the excavation. This allows for the calculation of the excavation disturbed or **induced** stresses.

Brady & Brown (2006)
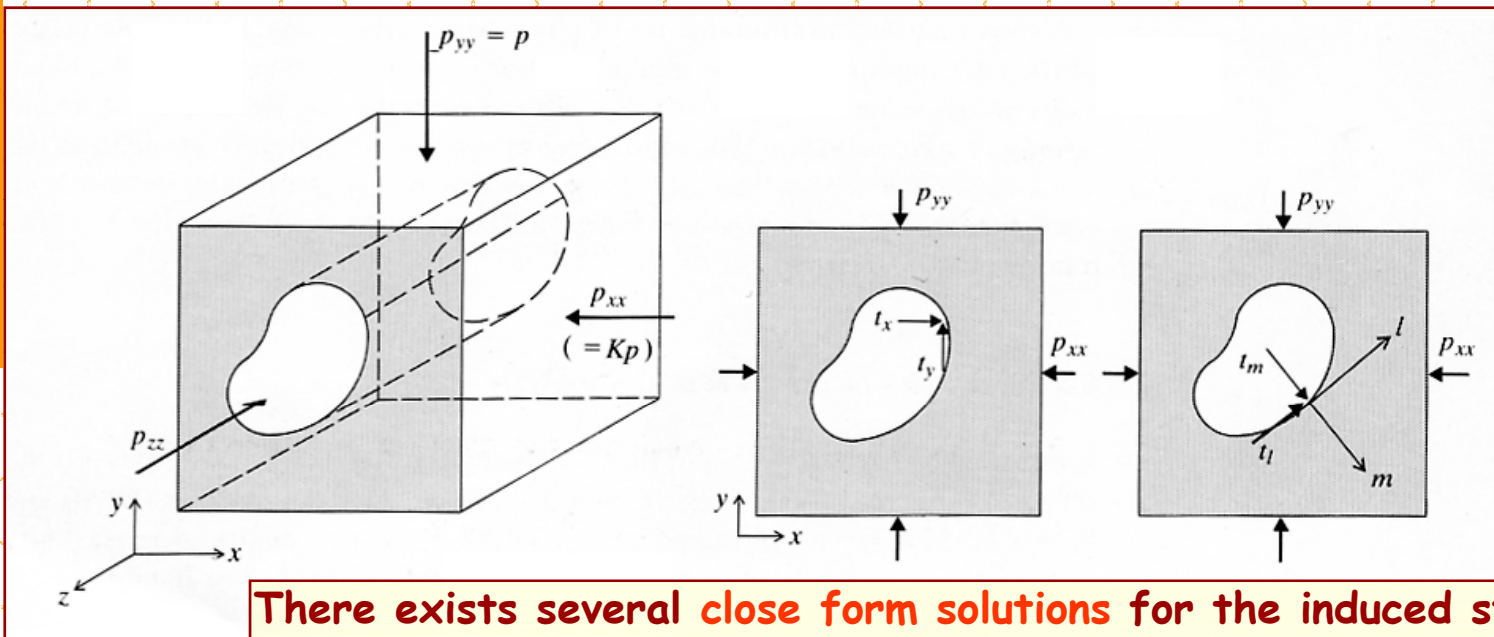
There exists several **close form solutions** for the induced stresses around circular and elliptical openings (and complex variable techniques extend these to many smooth, symmetrical geometries), and with **numerical analysis techniques** the values of the induced stresses can be determined accurately for any three-dimensional excavation geometry.

Stresses & Displacements - Circular Excavations

The Kirsch equations are a set of closed-form solutions, derived from the theory of elasticity, used to calculate the stresses and displacements around a circular excavation.
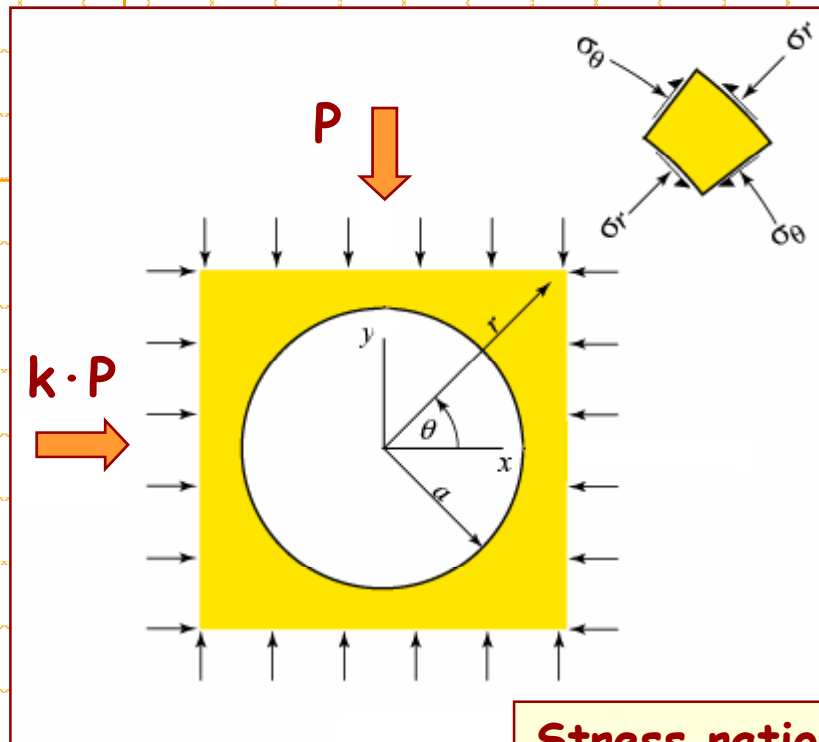
$$\sigma_{rr} = \frac{p}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right) - (1-K)\left(1-4\frac{a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{\theta\theta} = \frac{p}{2}\left[(1+K)\left(1+\frac{a^2}{r^2}\right) + (1-K)\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{r\theta} = \frac{p}{2}\left[(1-K)\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta\right]$$

$$u_r = -\frac{pa^2}{4Gr}\left[(1+K) - (1-K)\left\{4(1-\nu)-\frac{a^2}{r^2}\right\}\cos 2\theta\right]$$

$$u_\theta = -\frac{pa^2}{4Gr}\left[(1-K)\left\{2(1-2\nu)+\frac{a^2}{r^2}\right\}\sin 2\theta\right]$$

Brady & Brown (2006)

Stress ratio:
$$k = \sigma_h/\sigma_v$$
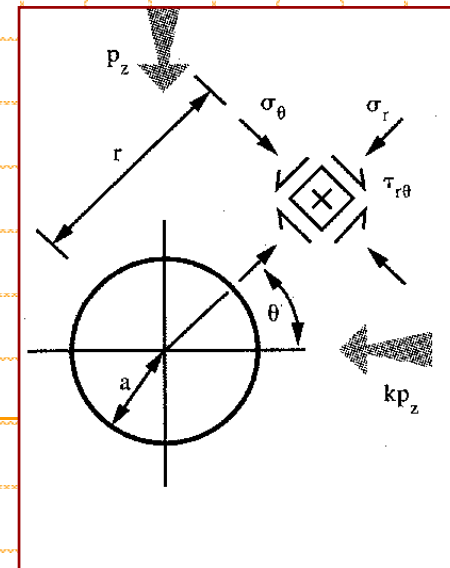
Stresses & Displacements - Circular Excavations

From these equations we can see that the stresses on the boundary (i.e. when r = a) are given by:

$$\sigma_{\theta\theta} = p[(1+k) + 2(1-k)\cos2\theta]$$

$$\sigma_{rr} = 0$$

$$\tau_{r\theta} = 0$$

Note that the radial stresses are zero because there is no internal pressure, and the shear stresses must be zero at a traction-free boundary.

Example 1: Stresses around a Circular Opening

Q. At a depth of 750 m, a 10-m diameter circular tunnel is driven in rock having a unit weight of 26 kN/m³ and uniaxial compressive and tensile strengths of 80.0 MPa and 3.0 MPa, respectively. Will the strength of the rock on the tunnel boundary be exceeded if:

    (a) k=0.3, and
    (b) k=2.0?
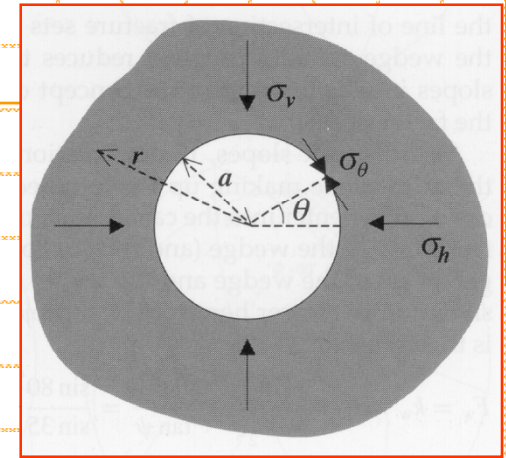
Harrison & Hudson (2000)

A. Since the tunnel has neither a support pressure nor an internal pressure applied to it, the local stresses at the boundary have $\sigma_3 = \sigma_r = 0$ and $\sigma_1 = \sigma_\theta$. The Kirsch solution for the circumferential stress is:

$$\sigma_\theta = \tfrac{1}{2}\sigma_v\left[(1+k)\left(1+\frac{a^2}{r^2}\right) + (1-k)\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

For a location on the tunnel boundary (i.e. a = r), this simplifies to:

$$\sigma_\theta = \sigma_v\left[(1+k) + 2(1-k)\cos 2\theta\right]$$

Example 1: Stresses around a Circular Opening

① First, we can assume that the vertical stress is equal to the weight of the overburden, giving:

$$\sigma_v = \gamma z = 0.026 \times 750 = 19.5 \, MPa$$
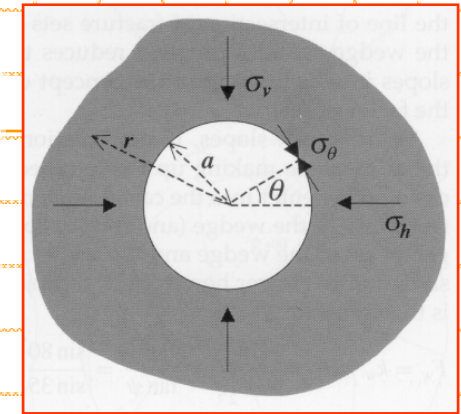
② The extreme values of induced stress occur at positions aligned with the principal *in situ* stresses, and so in order to compute the stress induced in the crown and invert (i.e. roof and floor) we use θ = 90°, and for the sidewalls we use θ = 0°.

For k=0.3:   Crown/invert (θ = 90°): σ_θ = -1.95 MPa (i.e. tensile)
             Sidewalls (θ = 0°): σ_θ = 52.7 MPa

For k=2.0:   Crown/invert (θ = 90°): σ_θ = 97.5 MPa   *compressive strength is exceeded*
             Sidewalls (θ = 0°): σ_θ = 19.5 MPa

Stress and Failure Criterion

A — develop excavation design to satisfy duty requirements

determine boundary stresses

compare $\sigma_{\theta\theta}$ with C_0, T_0
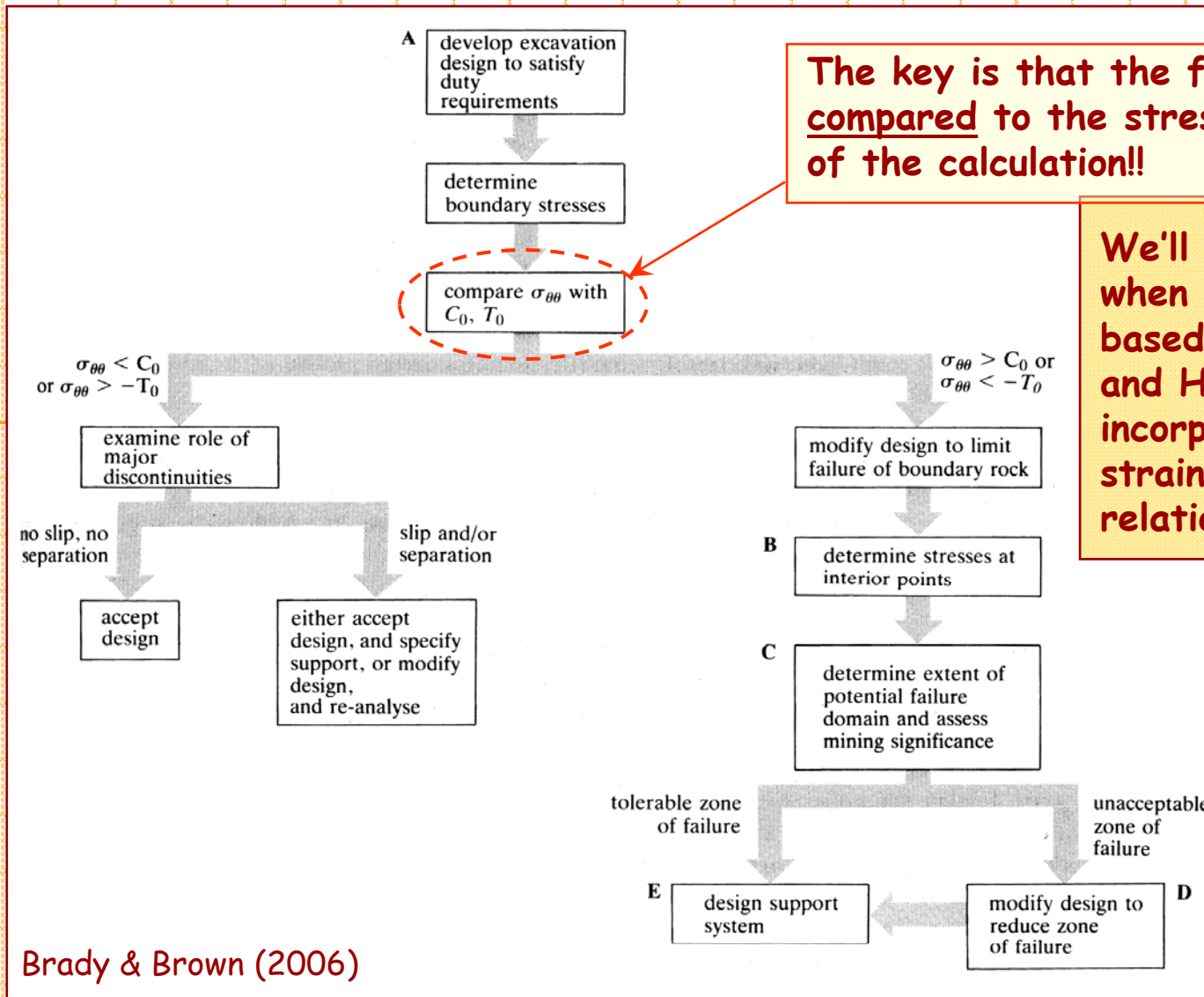
The key is that the failure criterion is **compared** to the stresses and is not part of the calculation!!

We'll see this is different when similar relationships based on Mohr-Coulomb and Hoek-Brown are incorporated into stress-strain constitutive relationships.

$\sigma_{\theta\theta} < C_0$ or $\sigma_{\theta\theta} > -T_0$

$\sigma_{\theta\theta} > C_0$ or $\sigma_{\theta\theta} < -T_0$

examine role of major discontinuities

modify design to limit failure of boundary rock

no slip, no separation

slip and/or separation

accept design

either accept design, and specify support, or modify design, and re-analyse

B — determine stresses at interior points

C — determine extent of potential failure domain and assess mining significance

tolerable zone of failure

unacceptable zone of failure

E — design support system

D — modify design to reduce zone of failure

Brady & Brown (2006)

Orientation of σ_1 & Induced Stresses
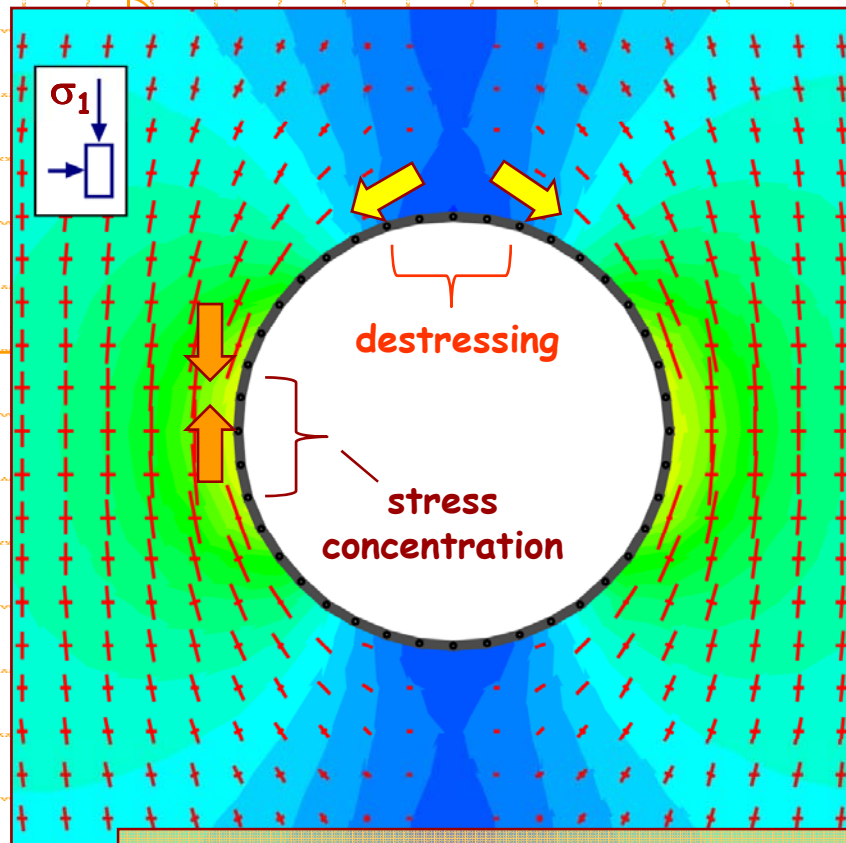
Potential Ground Control Issues:

Destressing = wedge failures
Concentration = spalling

σ_1

destressing

stress
concentration

σ_1

stress
concentration
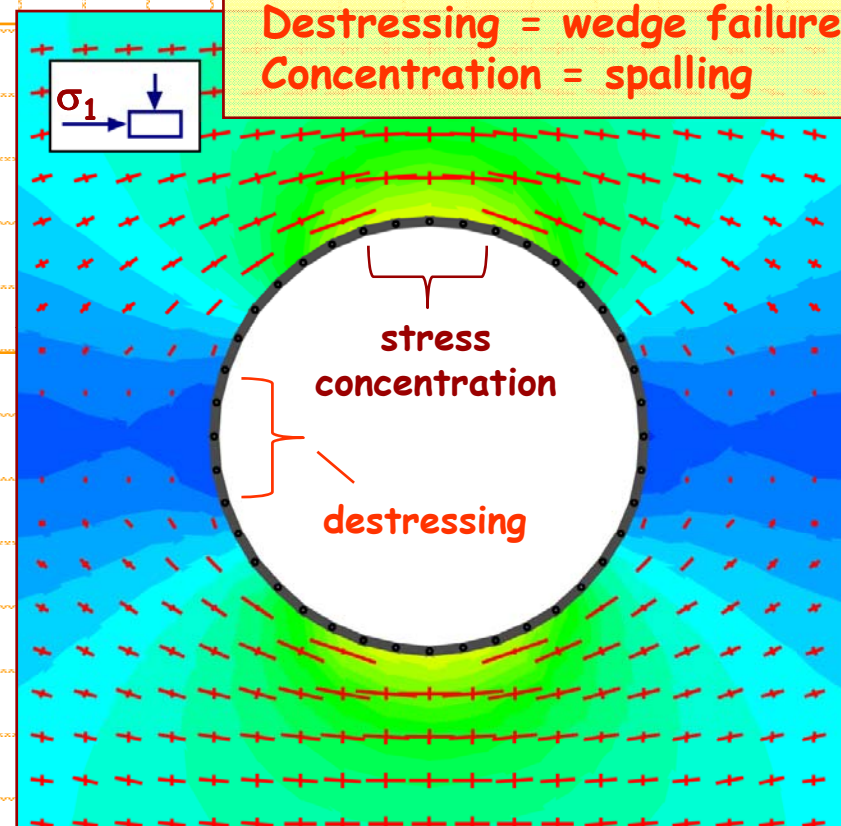
destressing

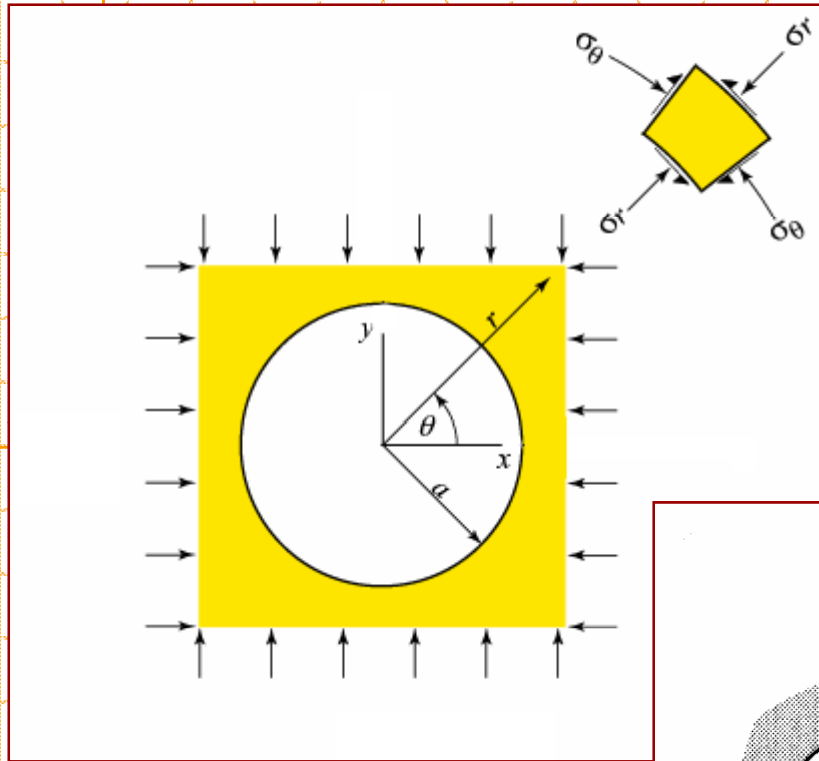Stresses can be visualized as flowing around the excavation periphery in the direction of the major principle stress (σ_1). Where they diverge, relaxation occurs; where they converge, stress increases occur.

Stresses Away from Opening

$$\sigma_r = \tfrac{1}{2}p_z \left\{ (1+k)\left(1 - \frac{a^2}{r^2}\right) - (1-k)\left(1 - 4\frac{a^2}{r^2} + 3\frac{a^4}{r^4}\right)\cos 2\theta \right\}$$

$$\sigma_\theta = \tfrac{1}{2}p_z \left\{ (1+k)\left(1 + \frac{a^2}{r^2}\right) + (1-k)\left(1 + 3\frac{a^4}{r^4}\right)\cos 2\theta \right\}$$

$$\tau_{r\theta} = \tfrac{1}{2}p_z \left\{ (1-k)\left(1 + 2\frac{a^2}{r^2} - 3\frac{a^4}{r^4}\right)\sin 2\theta \right\}$$

Brady & Brown (2006)

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Zone of Influence

The concept of influence is important in excavation design, since the presence of a neighbouring opening may provide a significant disturbance to the near-field stresses to the point of causing failure.

… (a) axisymmetric stress distribution around a circular opening in a hydrostatic stress field; (b) circular openings in a hydrostatic stress field, effectively isolated by virtue of their exclusion from each other's zone of influence.
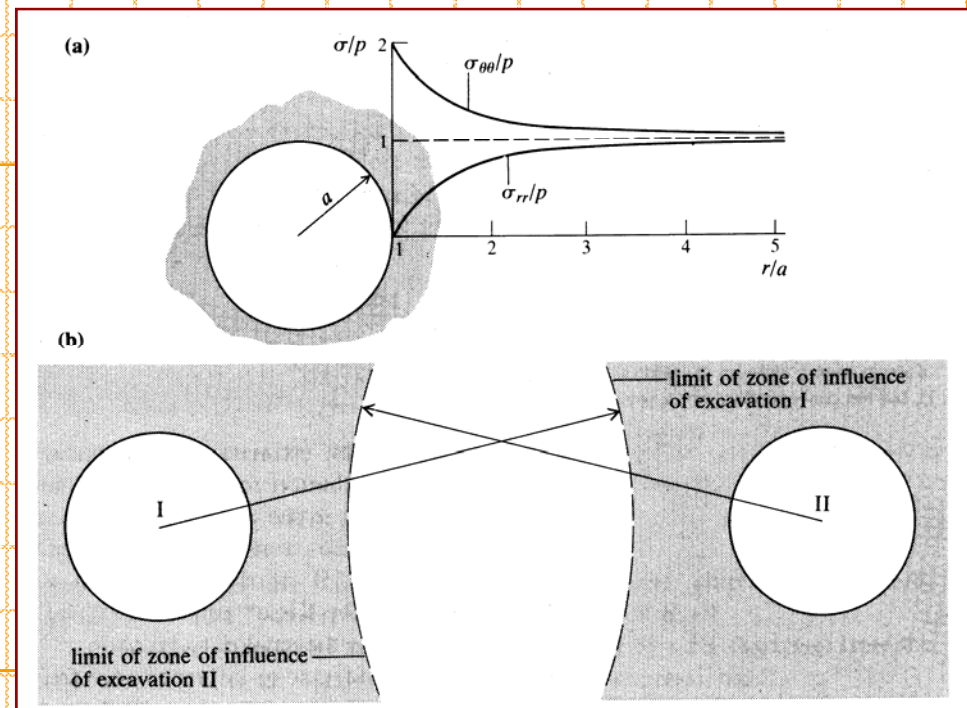
Brady & Brown (2006)

Zone of Influence

limit of zone of influence of excavation I

limit of zone of influence of excavation II
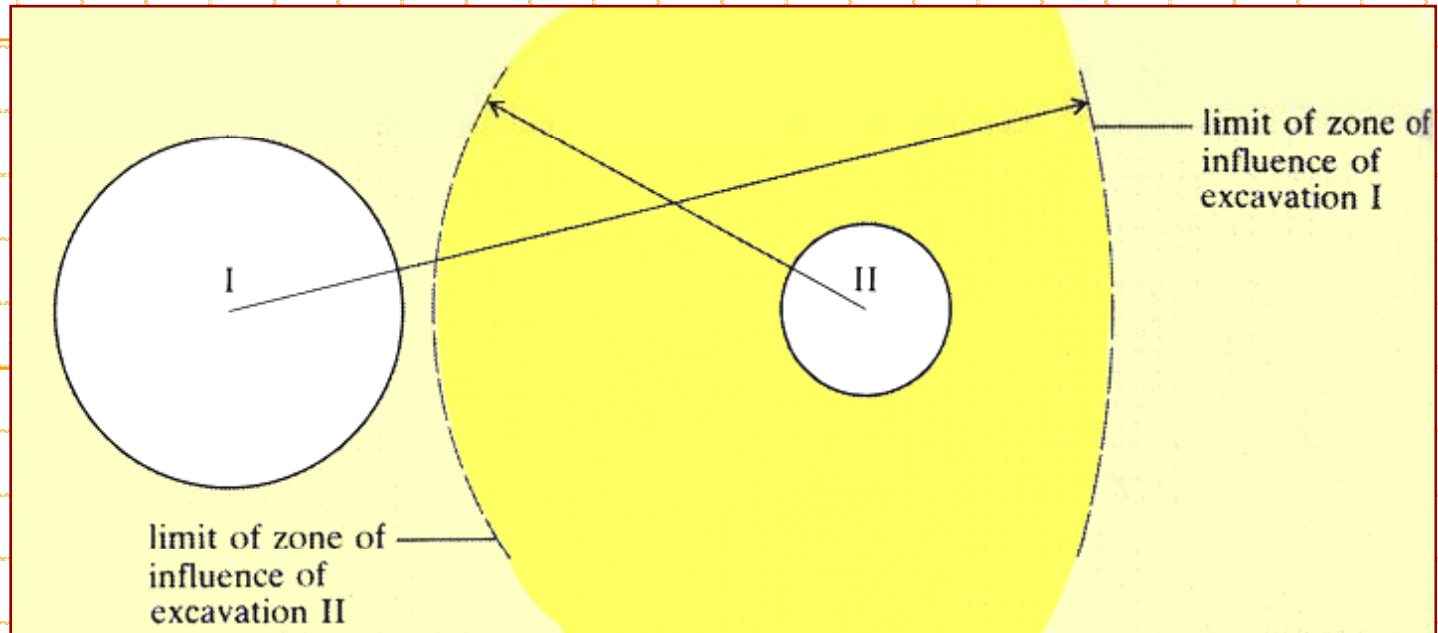
Brady & Brown (2006)

… illustration of the effect of contiguous openings of different dimensions. The zone of influence of excavation I includes excavation II, but the converse does not apply.

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Stresses Around Elliptical Openings

The stresses around **elliptical openings** can be treated in an analogous way to that just presented for circular openings. There is much **greater utility** associated with the solution for elliptical openings than circular openings, because these can provide a first approximation to a wide range of **engineering geometries**, especially openings with high width/height ratios (e.g. mine stopes, power house caverns, etc.).

From a design point of view, the effects of changing either the **orientation within the stress field** or the **aspect ratio** of such elliptical openings can be studied to optimize stability.

Stresses Around Elliptical Openings

Assuming isotropic rock conditions, an elliptical opening is completely characterized by two parameters: aspect ratio (major to minor axis) which is the eccentricity of the ellipse; and orientation with respect to the principle stresses. The position on the boundary, with reference to the x-axis, is given by the angle χ.

$$\sigma_\theta = \frac{p}{2q}\left\{(1+k)\left[(1+q^2)+(1-q^2)\cos 2\,(\chi-\beta)\right]\right.$$

$$\left.-(1-k)\left[(1+q)^2\cos 2\chi+(1-q^2)\cos 2\beta\right]\right\}$$

where $q = \dfrac{W}{H}$
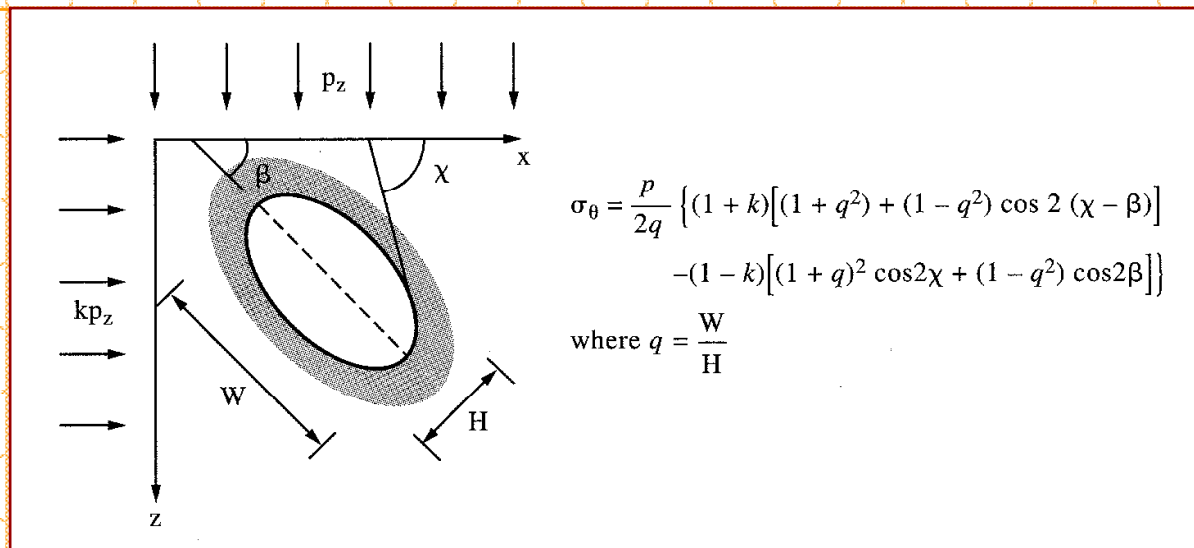
Hudson & Harrison (1997)

Stresses Around Elliptical Openings

It is instructive to consider the maximum and minimum values of the stress concentrations around the ellipse for the geometry of an ellipse aligned with the principal stresses. It can be easily established that the extremes of stress concentration occur at the ends of the major and minor axes.

$$\sigma_A = p(1 - k + 2q) = p\left(1 - k + \sqrt{\frac{2W}{\rho_A}}\right),$$

$$\sigma_B = p\left(k - 1 + \frac{2k}{q}\right) = p\left(k - 1 + k\sqrt{\frac{2H}{\rho_B}}\right)$$

where, for an ellipse, the radii of curvature are

$$\rho_A = \frac{H^2}{2W} \text{ and } \rho_B = \frac{W^2}{2H}$$
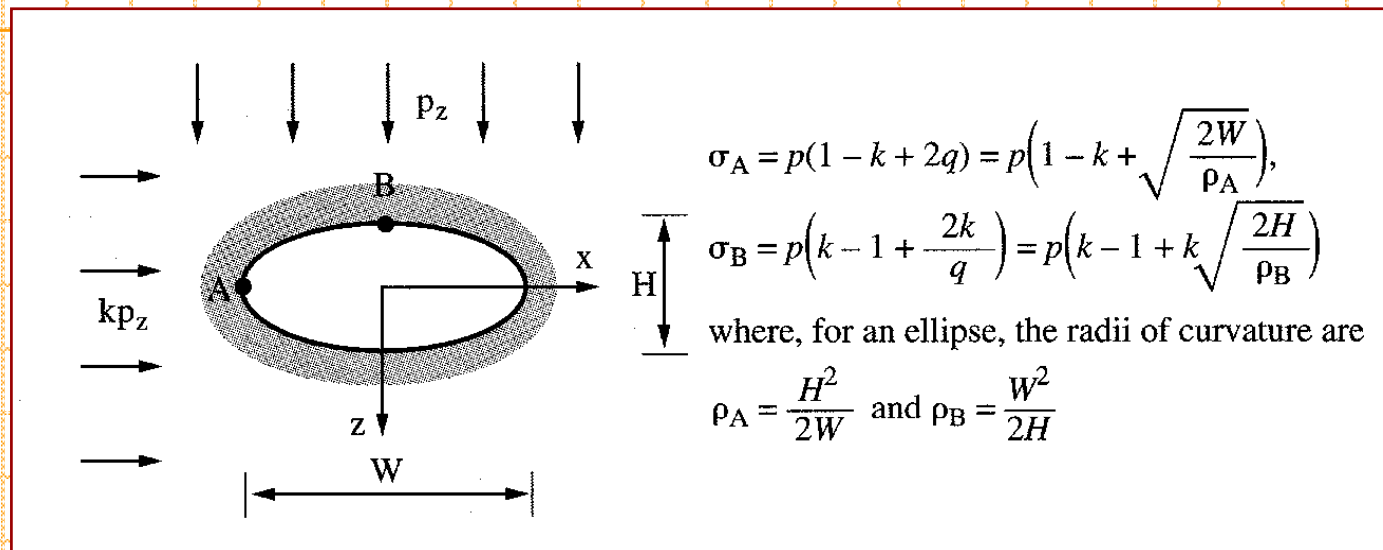
Hudson & Harrison (1997)

Example 2: Stresses around an Elliptical Opening

Q. A gold-bearing quartz vein, 2 m thick and dipping 90°, is to be exploited by a small cut-and-fill stoping operation. The mining is to take place at a depth of 800 m, and the average unit weight of the granite host rock above this level is 29 kN/m3. The strike of the vein is parallel to the intermediate stress, and the major principal stress is horizontal with a magnitude of 37.0 MPa. The uniaxial compressive strength of the vein material is 218 MPa, and the tensile strength of the host rock is -5 MPa. What is the maximum permissible stope height before failure occurs.

A. We can assume that, in 2-D cross-section, the stresses induced in the sidewalls (tensile) and the crown (compressive) of the stope can be approximated using the equations for an elliptical excavation.

$$\frac{\sigma_{\text{sidewall}}}{\sigma_{\text{vertical}}} = 1 - k + 2\left(\frac{w}{h}\right) \qquad \frac{\sigma_{\text{crown}}}{\sigma_{\text{vertical}}} = k - 1 + k\sqrt{\frac{2h}{\rho_{\text{crown}}}} = k - 1 + 2k\sqrt{\frac{h}{w}}.$$
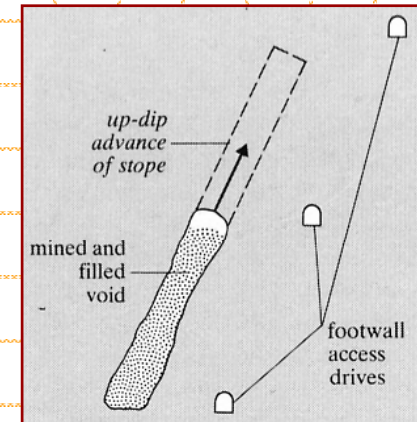
Harrison & Hudson (2000)

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Example 2: Stresses around an Elliptical Opening

① **Rearranging the given equations, we can solve for the height of the excavation as the minimum of:**

$$h = \frac{2w}{\dfrac{\sigma_{\text{sidewall}}}{\sigma_{\text{vertical}}} + k - 1} \quad \textbf{or} \quad h = \frac{w}{4k^2}\left(\frac{\sigma_{\text{crown}}}{\sigma_{\text{vertical}}} + 1 - k\right)^2$$

② **The maximum stress that can be sustained by the crown and the sidewall are 218 and –5 MPa, respectively. Note that the sidewall stress is negative because this represents the tensile strength.**

③ **The vertical stress is:** $\sigma_{\text{vertical}} = \gamma \cdot z = 0.029 \times 800 = 23.2\,\text{MPa}$

and hence the ratio of horizontal to vertical stress is: $k = \dfrac{37.0}{23.2} = 1.59$

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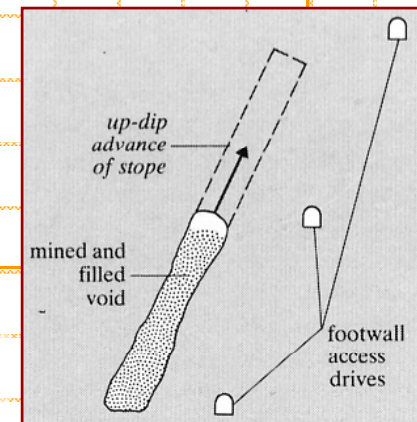# Example 2: Stresses around an Elliptical Opening

④

The maximal height of a stope such that the compressive strength of the rock in the crown is not exceeded is given by:

$$h = \frac{w}{4k^2}\left(\frac{\sigma_{\text{crown}}}{\sigma_{\text{vertical}}} + 1 - k\right)^2 = \frac{2}{4 \times 1.59^2}\left(\frac{218}{23.2} + 1 - 1.59\right)^2 = 15.3 \text{ m.}$$

The maximal height of a stope such that the tensile strength of the rock in the sidewall is not exceeded is given by:

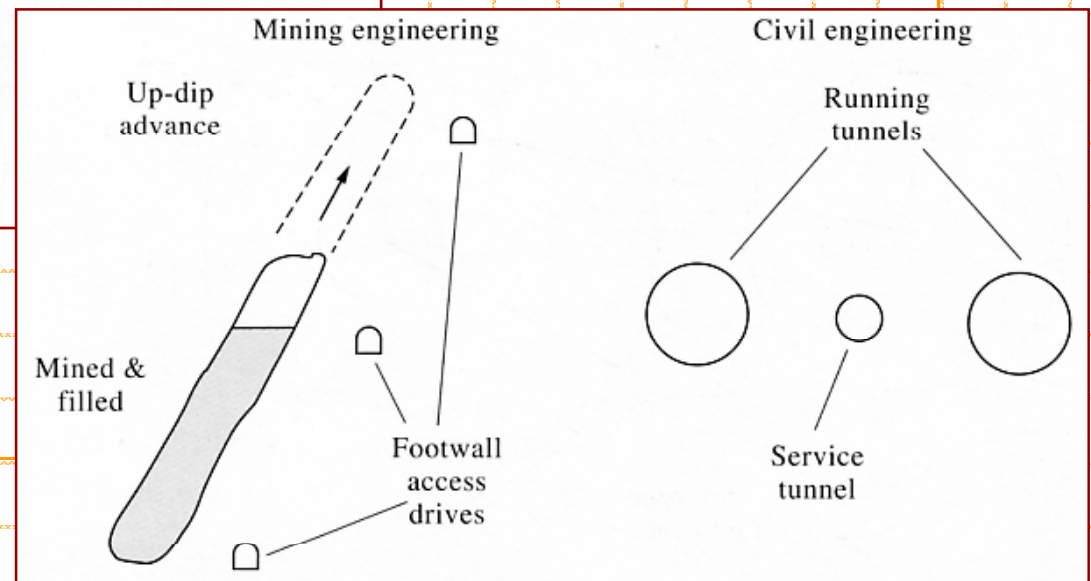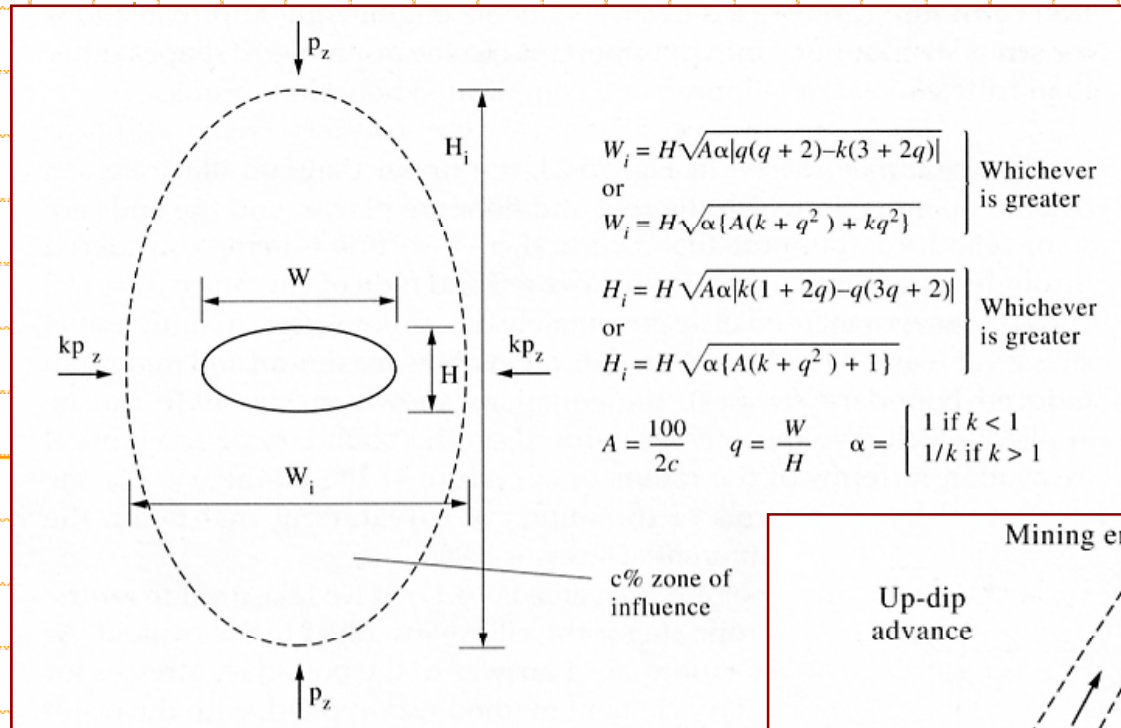$$h = \frac{2w}{\dfrac{\sigma_{\text{sidewall}}}{\sigma_{\text{vertical}}} + k - 1} = \frac{2 \times 2}{\dfrac{-5}{23.2} + 1.59 - 1} = 10.7 \text{ m.}$$

Thus we see that sidewall failure is the limiting condition if no stress-induced failure is acceptable in the stope design.

up-dip advance of stope

mined and filled void

footwall access drives

Harrison & Hudson (2000)

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Zone of Influence

$W_i = H \sqrt{A\alpha |q(q+2) - k(3+2q)|}$

or

$W_i = H \sqrt{\alpha \{A(k + q^2) + kq^2\}}$

Whichever is greater

$H_i = H \sqrt{A\alpha |k(1+2q) - q(3q+2)|}$

or

$H_i = H \sqrt{\alpha \{A(k + q^2) + 1\}}$

Whichever is greater

$A = \dfrac{100}{2c} \qquad q = \dfrac{W}{H} \qquad \alpha = \begin{cases} 1 \text{ if } k < 1 \\ 1/k \text{ if } k > 1 \end{cases}$

$c\%$ zone of influence

... elliptical approximation to the zone of influence around an elliptical excavation.

Mining engineering

Up-dip advance

Mined & filled

Footwall access drives

Civil engineering

Running tunnels

Service tunnel

Hudson & Harrison (1997)

Many underground excavations are **irregular** in shape and are frequently grouped close to **other excavations**. These problems require the use of **numerical techniques**.

Sill Pillar

direction of mining advancement

Legend (MPa) < 0   0   15   30   45   60   75   90   >90
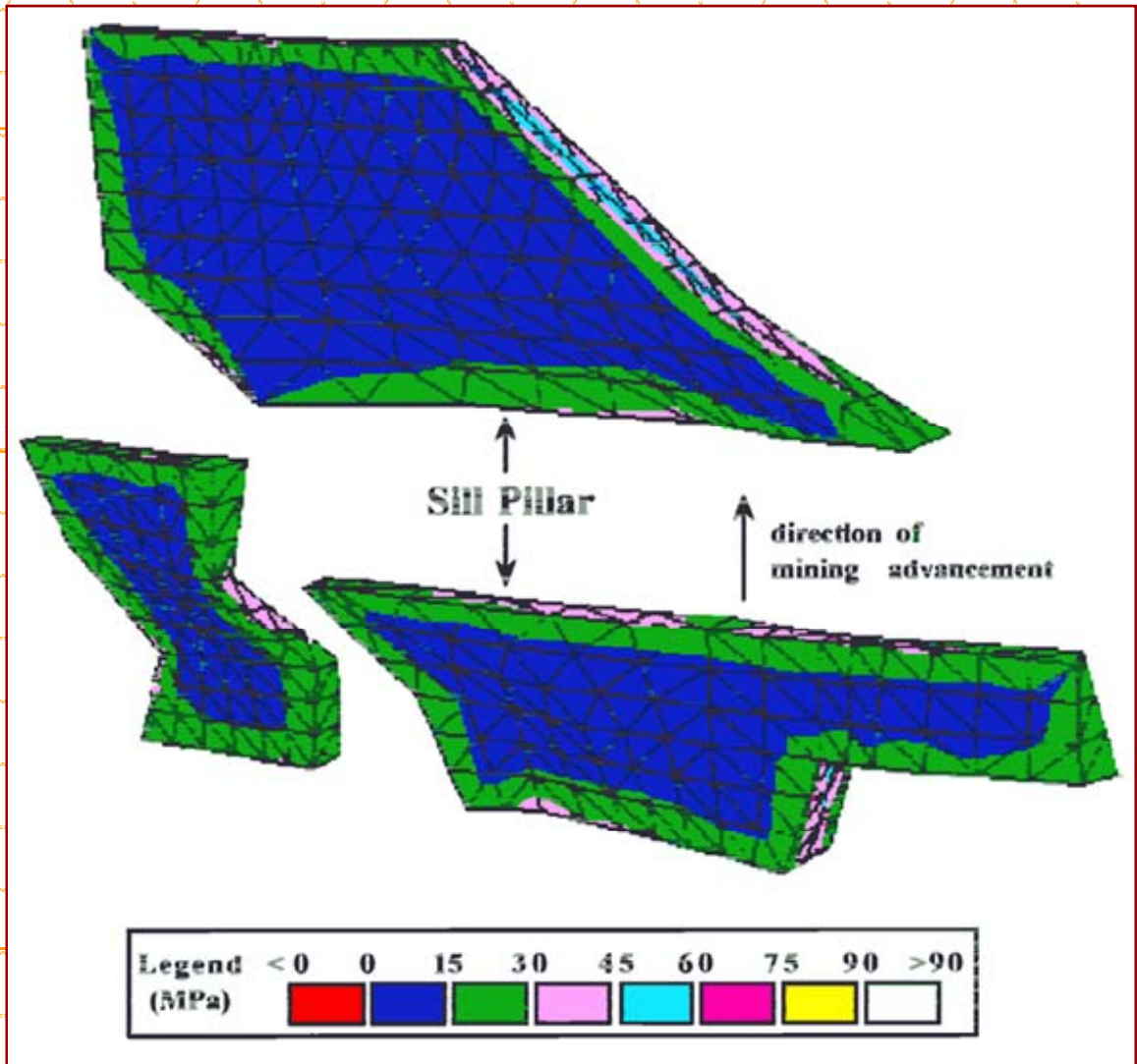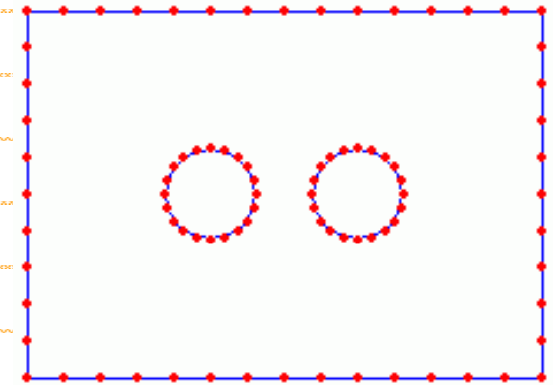
Eberhardt *et al.* (1997)

Numerical Modelling

Numerical methods of stress and deformation analysis fall into two categories:

Integral Methods

→ incl. boundary-element method

→ only problem boundary is defined & discretized

→ <u>Pro</u>: more computationally efficient
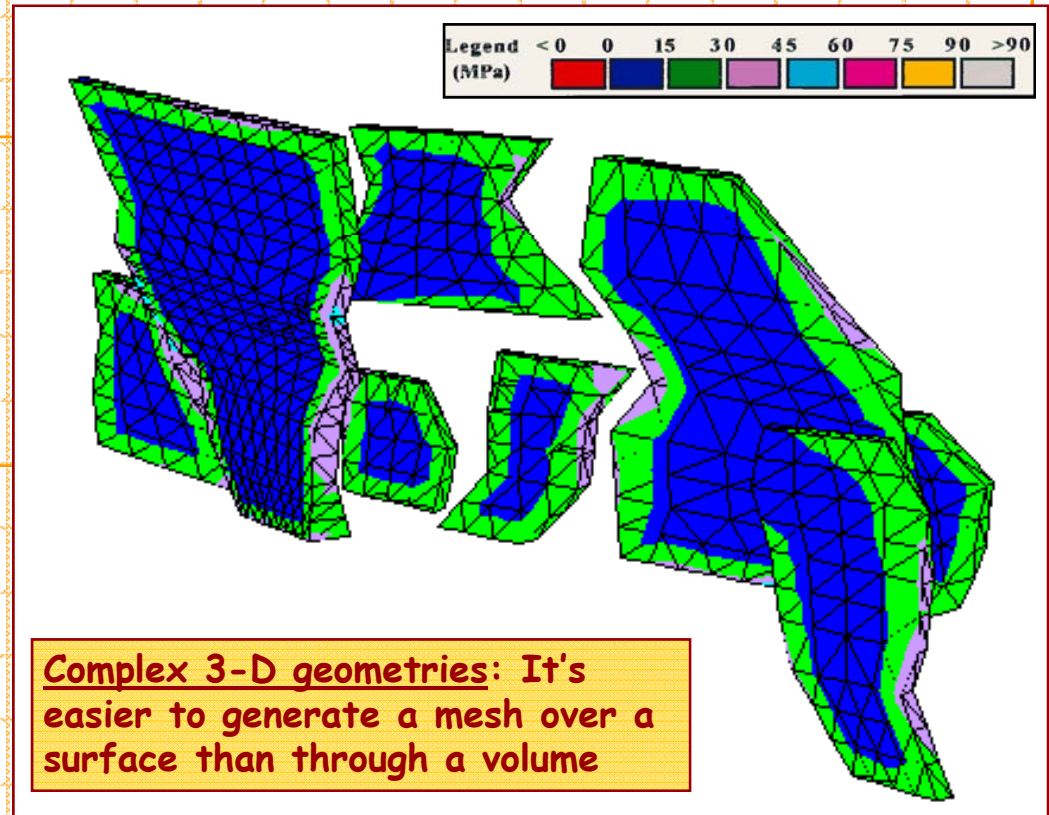<u>Con</u>: restricted to elastic analyses

Differential Methods

→ incl. finite-element/-difference & distinct-element methods

→ problem domain is defined & discretized

→ <u>Pro</u>: non-linear & heterogeneous material properties accommodated
<u>Con</u>: longer solution run times
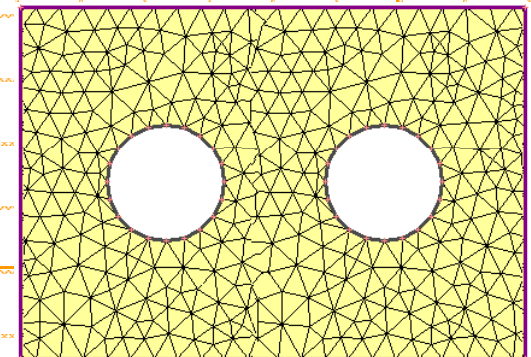
Boundary-Element & Stress Analyses

The Boundary Element Method (BEM) is generally favoured for stress analyses involving multiple excavations with complex 3-D geometries (e.g. those frequently encountered in underground mine design). The irregular shape of the orebodies and staged nature of mining, makes the ease of mesh generation and computational efficiency afforded by the BEM highly advantageous.


Legend (MPa): < 0   0   15   30   45   60   75   90   >90

Complex 3-D geometries: It's easier to generate a mesh over a surface than through a volume

Eberhardt *et al.* (1997)

Commercial Software:

Examine3D (Rocscience) - http://www.rocscience.com/
Map3D (Mine Modelling Pty Ltd.) - http://www.map3d.com/
BEFE (Computer Software & Services - CSS) - http://members.chello.at/sylvia.beer/
GPBEST (Best Corp.) - http://www.gpbest.com/
BEASY (Beasy Group) - http://www.beasy.com/

Boundary-Element & Stress Analyses

In performing an analysis, the boundary of the excavation is divided into elements and the interior of the rock mass is represented mathematically as an infinite continuum. The solution works to find a set of approximate stresses which satisfy prescribed boundary conditions, and then uses these to calculate the stresses and displacements in the rock mass.
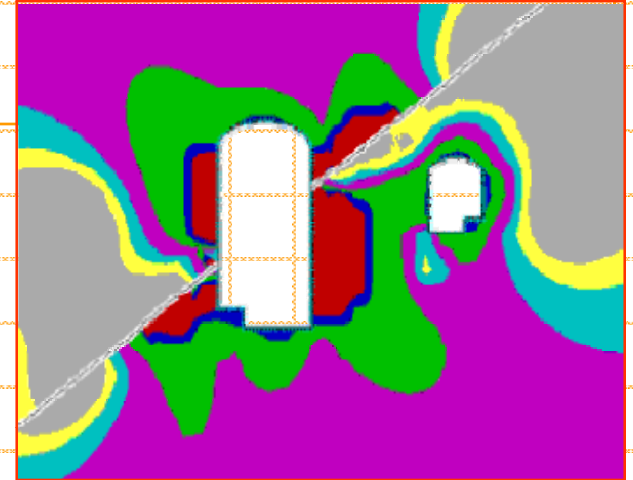
What to Know:

- Computational method of solving linear partial differential equations which have been formulated as integral equations (i.e. in boundary integral form).

- Key advantage is the reduction of the model dimension by one, providing simpler mesh generation & input data preparation, and greater computational efficiency.

- Key disadvantage is the required assumption of homogeneous linear elastic material behaviour; plasticity and heterogeneity negate the method's intrinsic simplicity.

- Common applications include stress analysis of underground excavations, soil-structure interactions, brittle fracturing processes, dynamic problems, groundwater flow and coupled H-M & T-H-M problems.

Erik Eberhardt – UBC Geological Engineering <u>ISRM Edition</u>
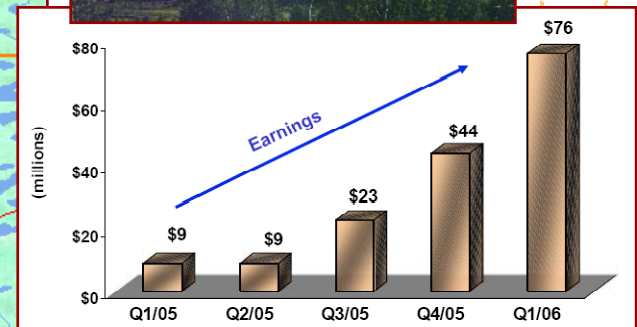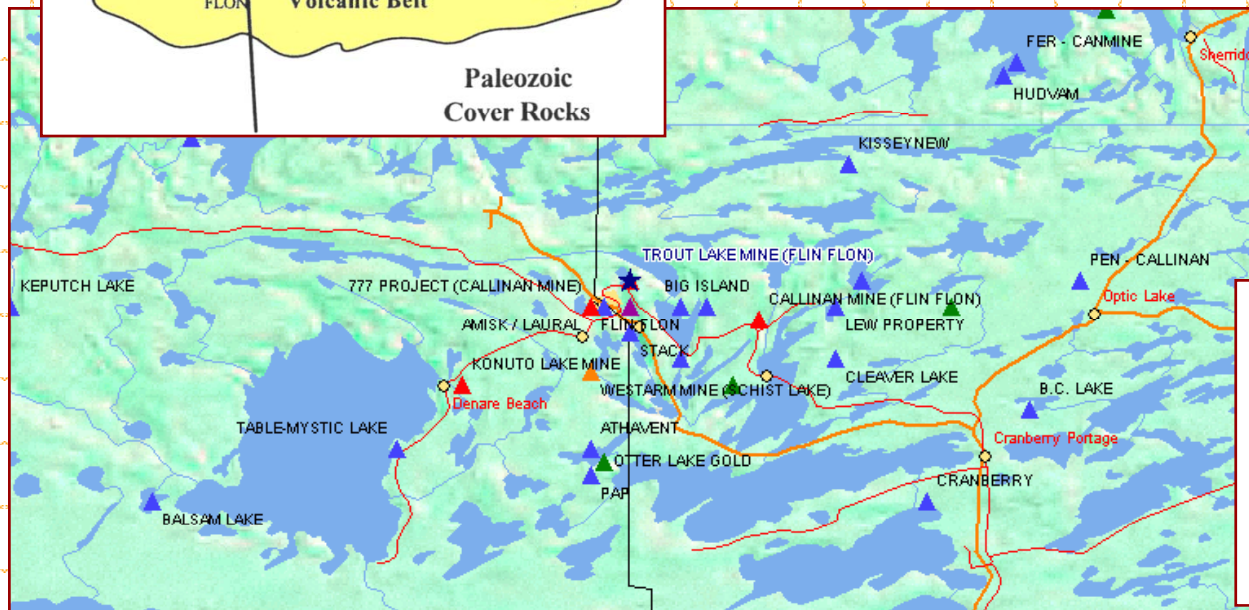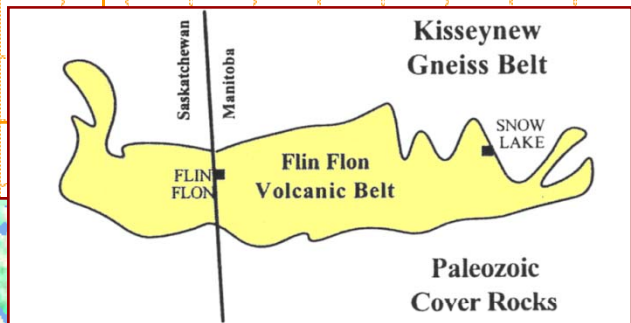
Boundary-Element & Stress Analysis
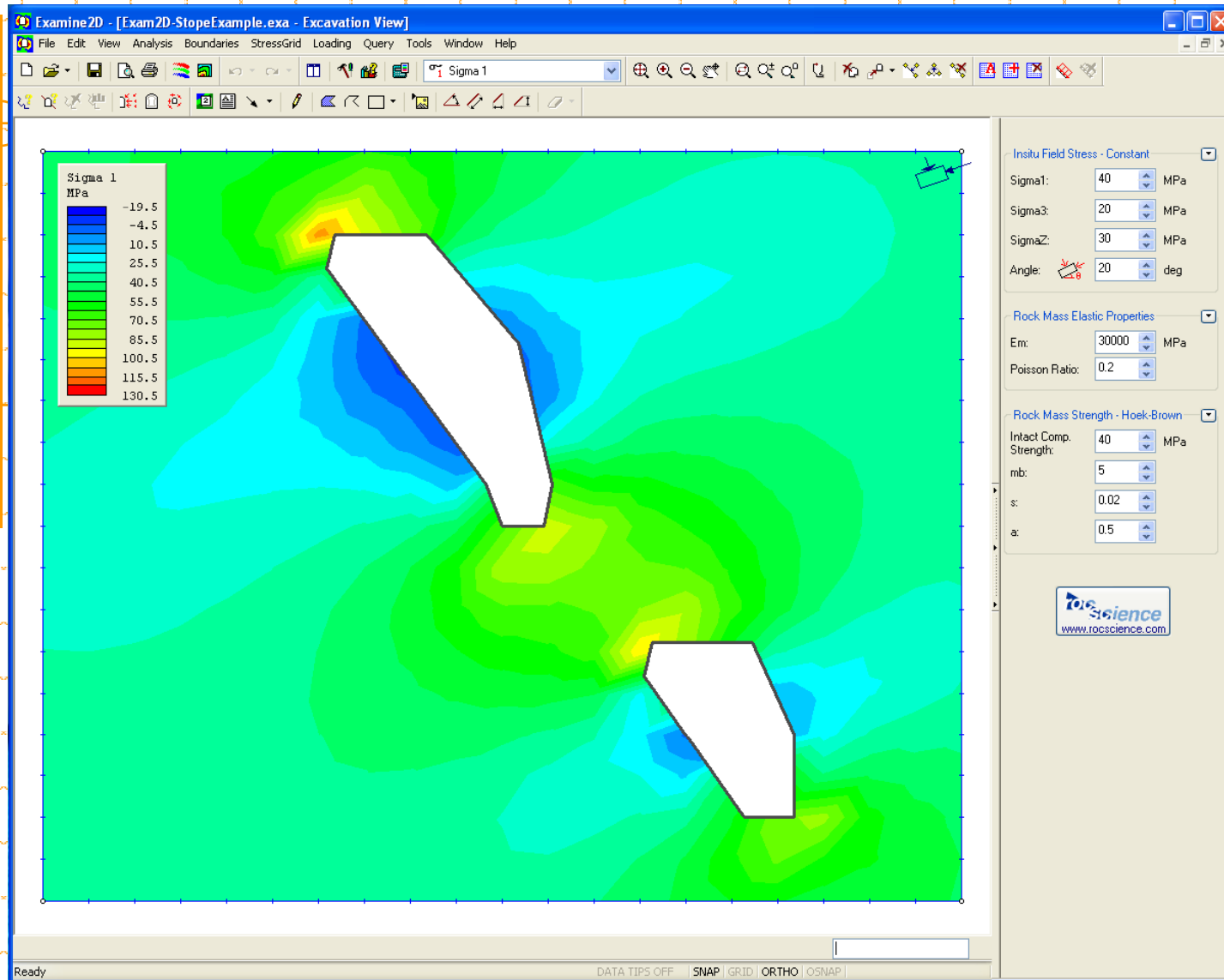
(Rocscience – Examine2D)

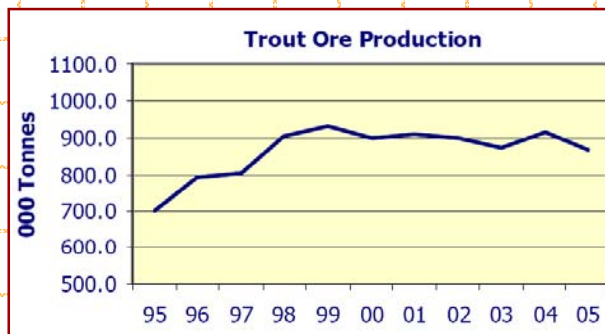# Case History: Trout Lake Mine, Flin Flon

Since 1927, Hudson Bay Mining & Smelting has developed more than 29 mines in the Flin Flon Greenstone belt, with continuous production of zinc, copper, silver, and gold since 1930.

Case History: Trout Lake Mine, Flin Flon

	Tonnes	Au (g/t)	Ag (g/t)	Cu (%)	Zn (%)
Trout Lake					
Proven	1,433,000	1.0	11.1	1.4	4.7
Probable	1,058,000	1.7	20.5	2.5	4.0

The Trout Lake Cu-Zn sulphide deposit was discovered in 1976, with mining beginning in 1982. The deposit is a VMS-type deposit that involves two ore zones 500 m apart, which reach depths exceeding 1200 m.

Flin Flon Greenstone Belt Mines

Discovery Method
- Geological (Red)
- Geophysical (Blue)
- Prospecting (Green)

Projected life of mine.

In a historic mining centre like Flin Flon, the life of the town is dependent on the ability to mine new and deeper reserves to feed the mill and keep the smelter running.

Discovery   Closure

Mines (top to bottom): Konuto, Photo, "777", Chisel North, Callinan, Namew, TROUT, Spruce Point, Westarm, Rod, Centennial, Ruttan, White, Anderson, Ghost & Lost, Stall, Chisel, Osborne, Coronation, Flexar, Birch, North Star, Don Jon, Schist, Cuprus, Dickstone, Sherridon, Flin Flon, Mandy

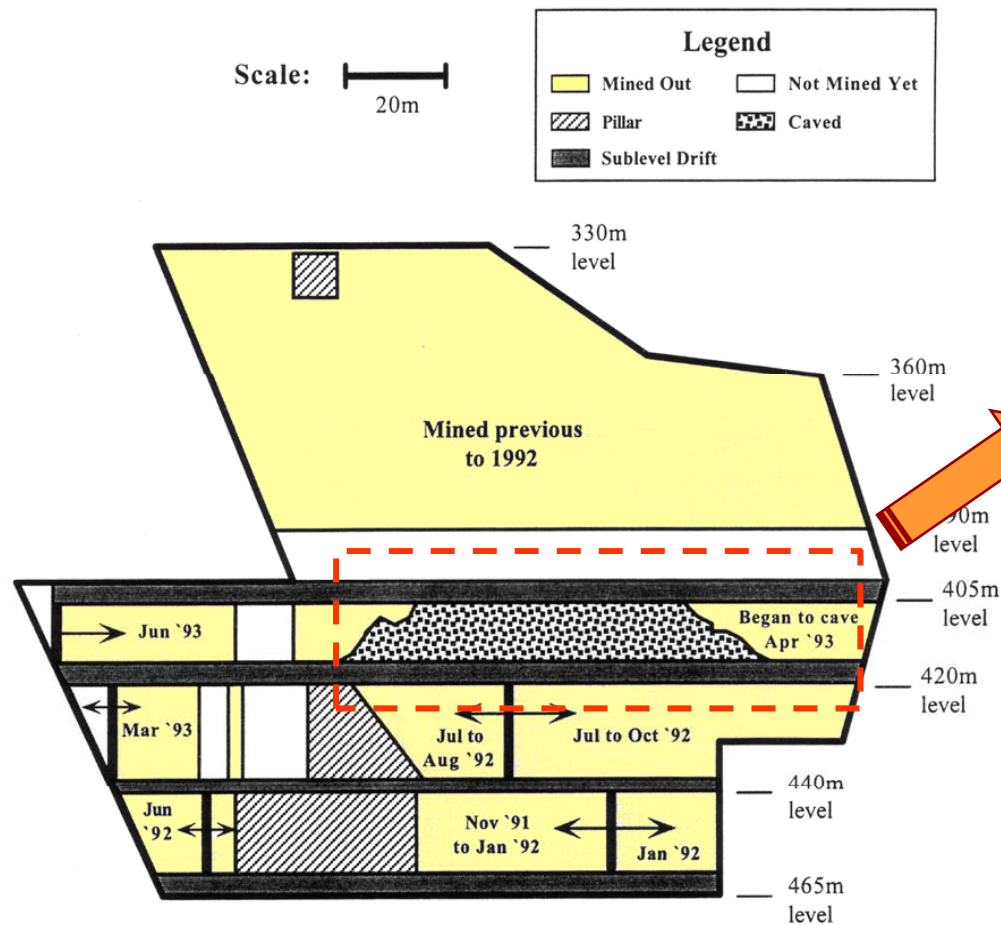Years: 1910, 1930, 1950, 1970, 1990, 2010

Initial mining was carried out by ramping down to the deposit and mining it at relatively shallow depths through sub-level stoping. To what depths, though, will mining be economical?
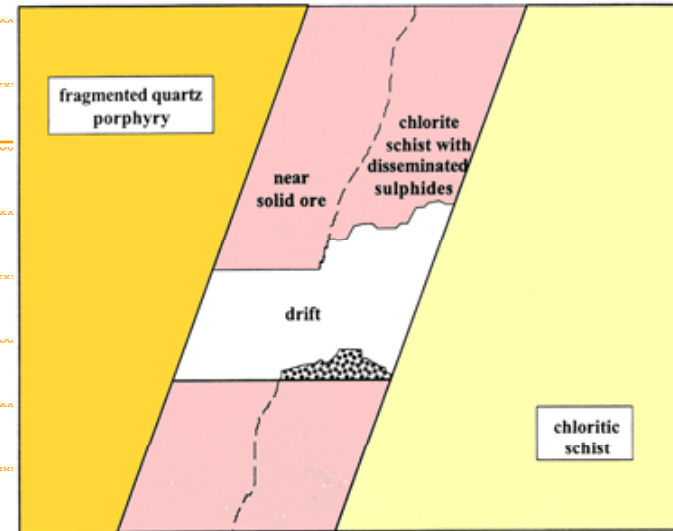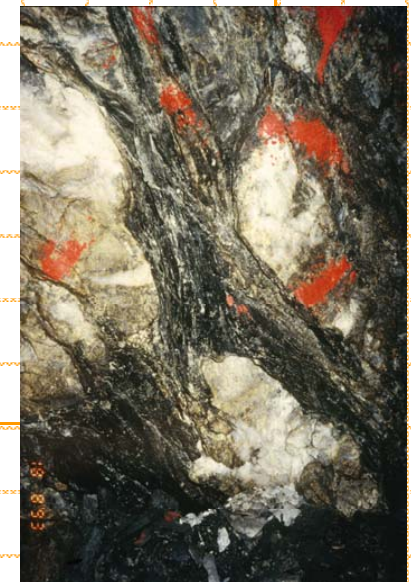

EMBURY (TROUT) LAKE

Mafic Flow

Hangingwall : Sericitized Quartz - Phyric Fragmental Rhyolite

Mafic Flow

No. 1 Lens

Altered Zone

Sericitized Quartz - Phyric Fragmental Rhyolite

Gabbro

Gabbro

Sericitized Quartz - Phyric Fragmental Rhyolite

100m Level

200m Level

300m Level

■ Solid Ore

▒ Disseminated Ore

No. 2 Lens

Altered Zone

Trout Lake Mine
North Zone

Cross Section   3+00E
Looking N130°

0 10 25   50   100
metres

headframe

overburden   surface production   fan

open pit

pillar (horizontal)   ventilation shaft

sublevel I   auxiliary level

stope   ore pass

sublevel II   ramp

haulage drift   main shaft

main level I

waste pass

OREBODY   ore pass

manway raise

main level II

foot wall   water basin

hanging wall   underground crusher

dip   pump station

skip

ore bin

core drilling   drift exploration   winze

skip filling station   sump

underground production

under development for production   exploration

Brady & Brown (2006)

Case History: Trout Lake Mine, Flin Flon

Eberhardt et al. (1997)

Legend

(yellow)	Mined Out	(white)	Not Mined Yet
(hatched)	Pillar	(dotted)	Caved
(dark)	Sublevel Drift		

Scale: 20m

330m level
360m level
390m level
405m level
420m level
440m level
465m level

Mined previous to 1992

Jun '93
Mar '93
Jun '92
Began to cave Apr '93
Jul to Aug '92
Jul to Oct '92
Nov '91 to Jan '92
Jan '92

fragmented quartz porphyry
near solid ore
chlorite schist with disseminated sulphides
drift
chloritic schist

TL004 - 2 - 3
1 cm

Ground control, dilution and drift stability problems encountered at 400m depth.
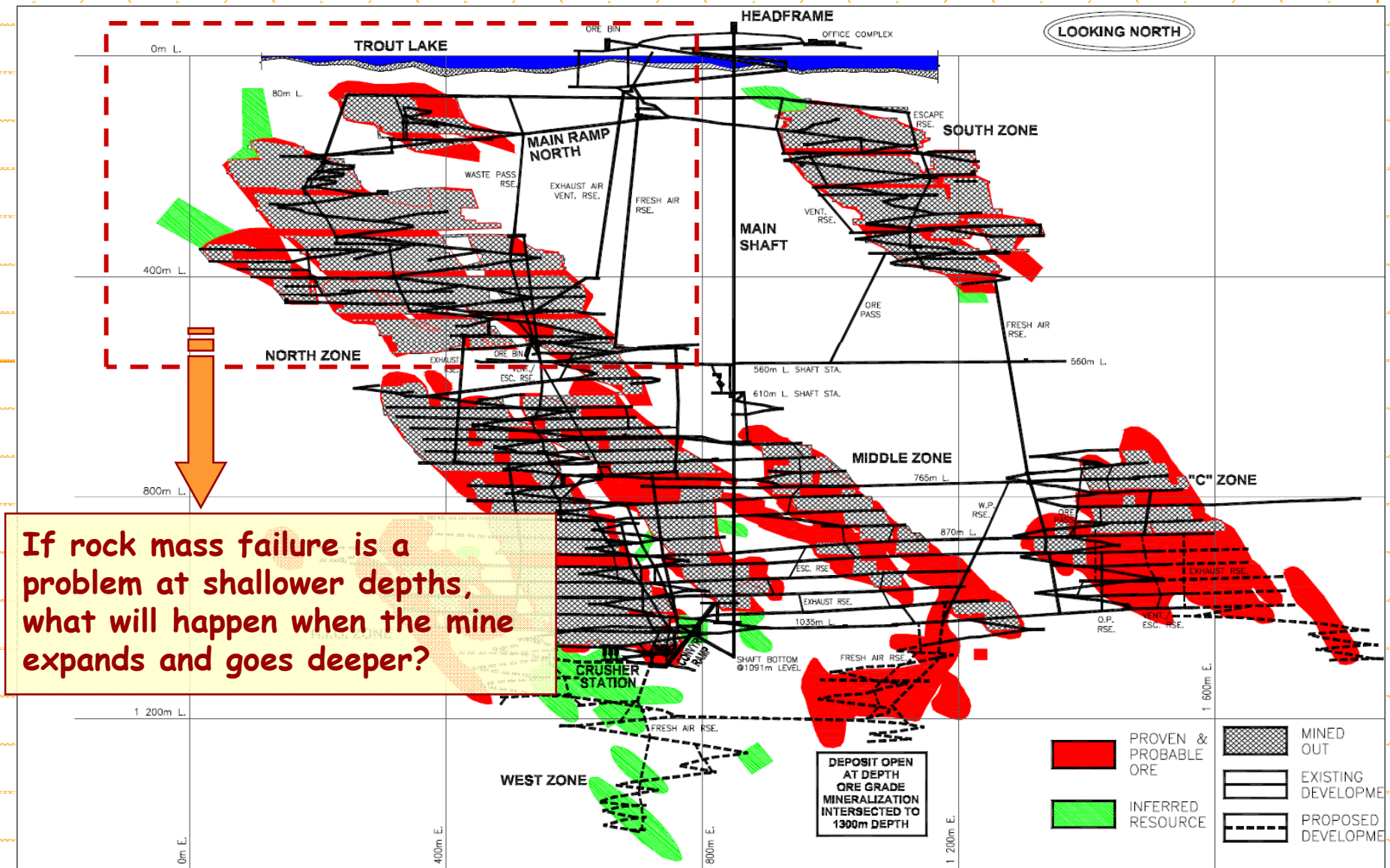
Eberhardt *et al.* (1997)

If rock mass failure is a problem at shallower depths, what will happen when the mine expands and goes deeper?
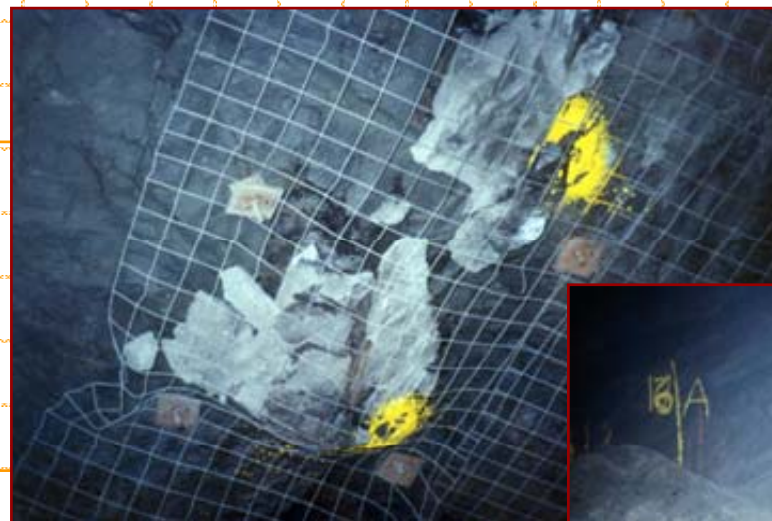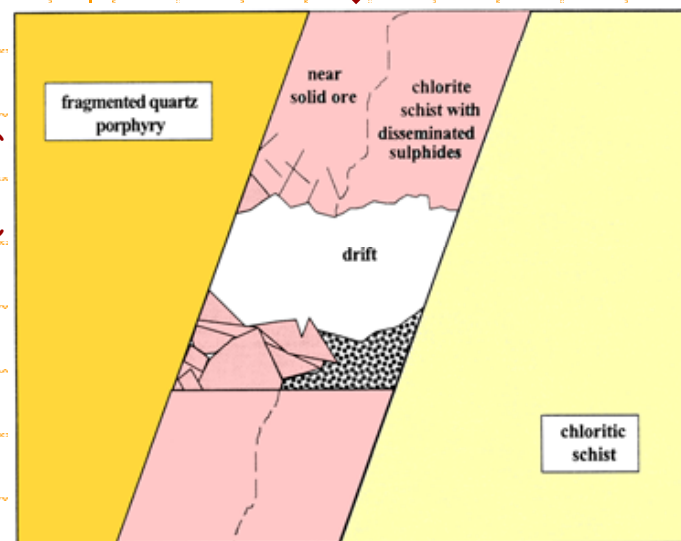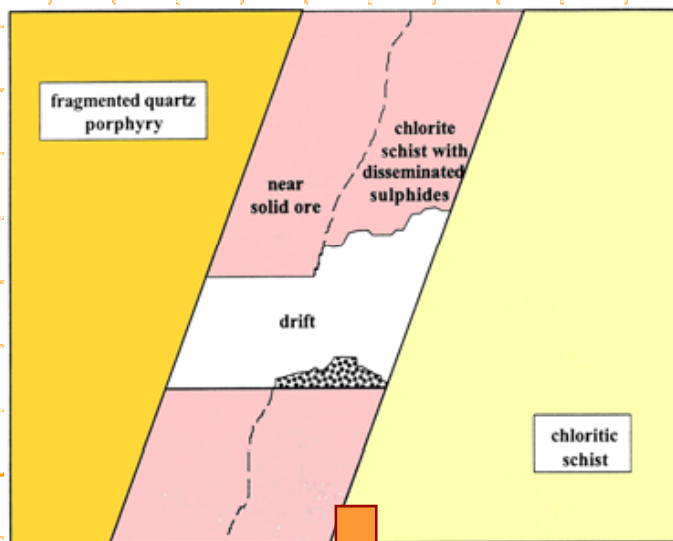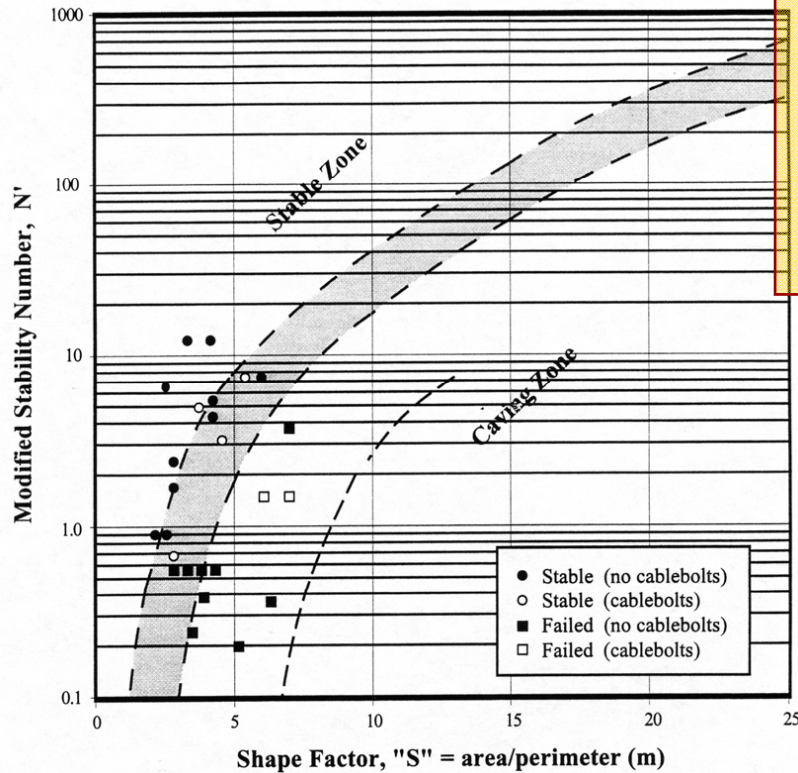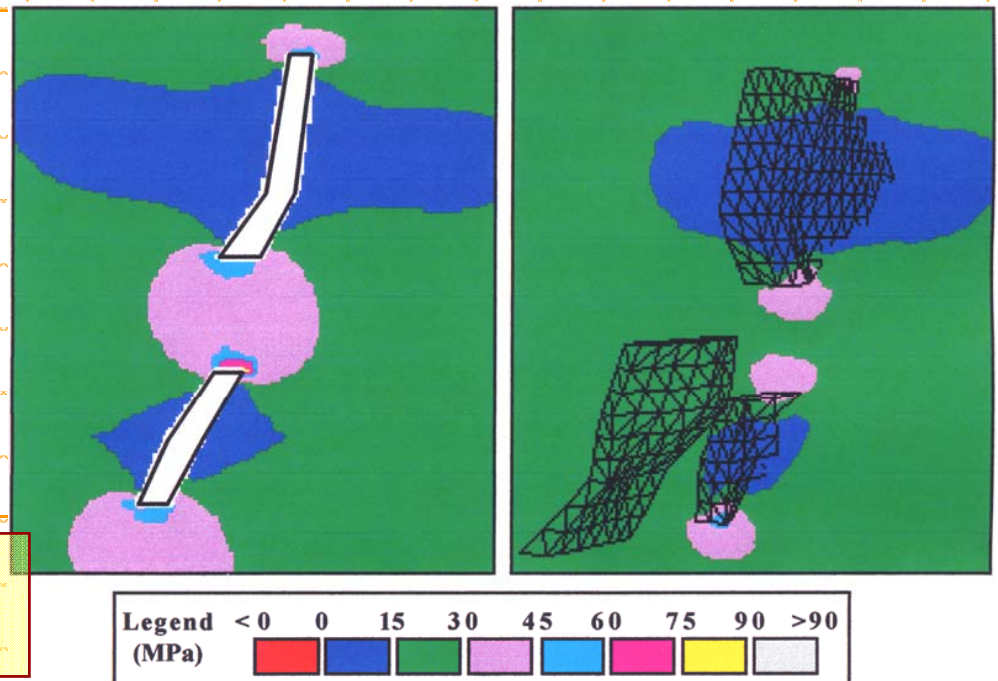
Case History: Trout Lake Mine Stress Analysis

Reschke & Romanowski (1993)

Empirical analysis based on performance of stope backs in the Flin Flon deposits, although useful, may be limited in its applicability if the database is populated with cases at shallower depths (i.e. lower stresses).
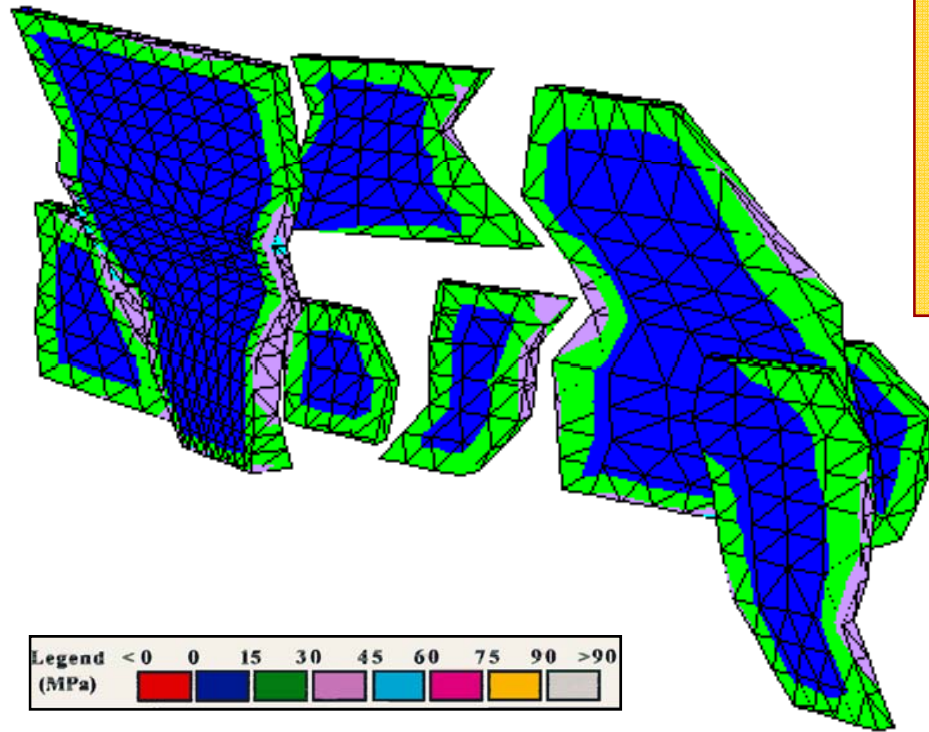
Eberhardt *et al.* (1997)

2-D and 3-D stress analyses of Trout Lake stopes.

Case History: Trout Lake Mine Stress Analysis

Because of the efficiency of the boundary element technique, 3-D analyses (e.g. those looking at the stress interactions between neighbouring stopes) becomes a relatively straightforward exercise.

Legend < 0   0   15   30   45   60   75   90   >90
(MPa)

Eberhardt *et al.* (1997)

Upper Stope

pillar

yield zone

stope back

Lower Stope

Legend   < 35   35   45   50   55   60   65   70   75   80   >85
(MPa)

Computed stresses can then be compared against estimated rock mass strengths to assess pillar stability.
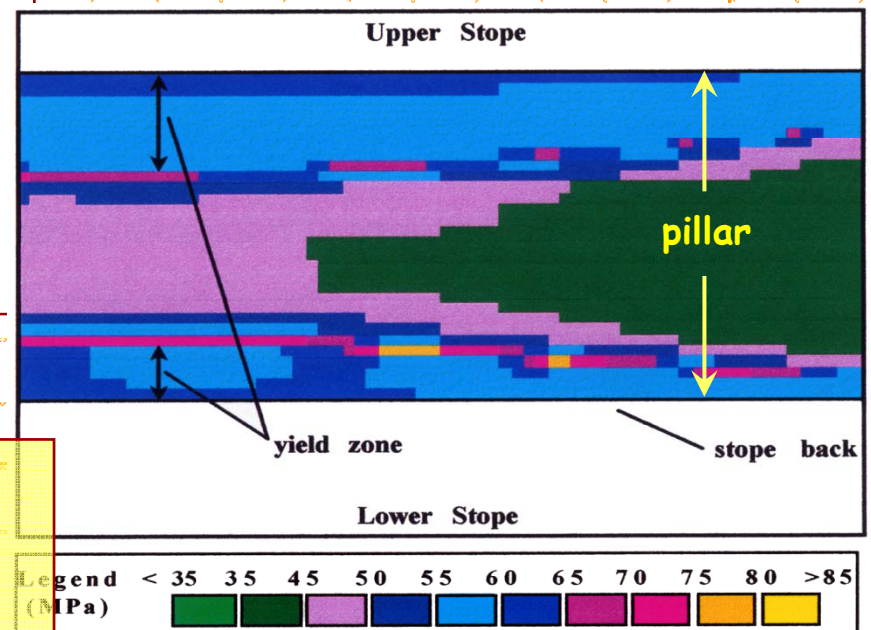
Lecture References

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