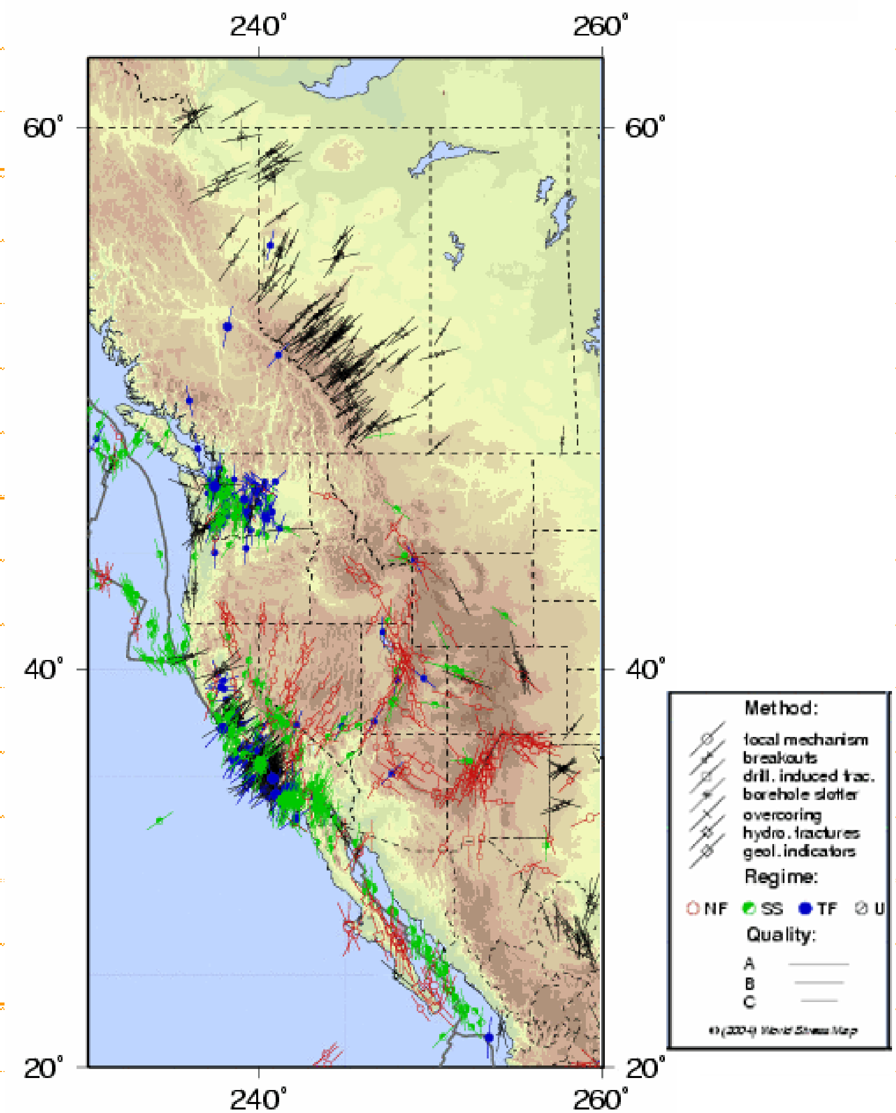
Rock Engineering Practice & Design

Lecture 7: *In Situ* Stresses & Stress Measurement



Author's Note:

The lecture slides provided here are taken from the course "Geotechnical Engineering Practice", which is part of the 4th year Geological Engineering program at the University of British Columbia (Vancouver, Canada). The course covers rock engineering and geotechnical design methodologies, building on those already taken by the students covering Introductory Rock Mechanics and Advanced Rock Mechanics.

Although the slides have been modified in part to add context, they of course are missing the detailed narrative that accompanies any lecture. It is also recognized that these lectures summarize, reproduce and build on the work of others for which gratitude is extended. Where possible, efforts have been made to acknowledge the various sources, with a list of references being provided at the end of each lecture.

Errors, omissions, comments, etc., can be forwarded to the author at: erik@eos.ubc.ca

Why Study Stress?

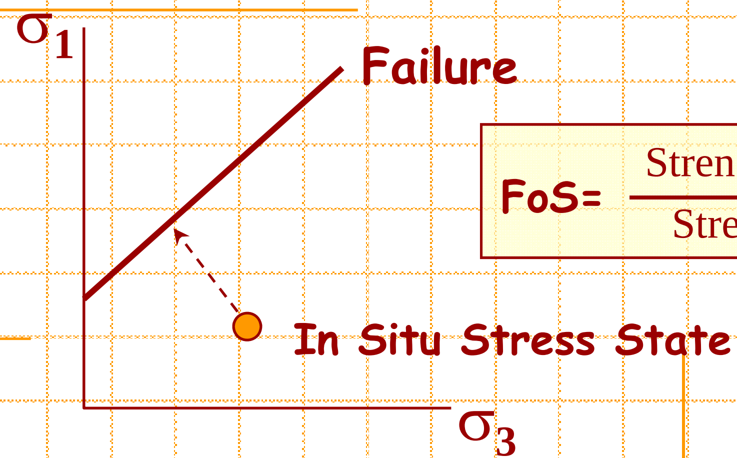
Stress is a concept which is fundamental to rock mechanics principles and applications. There are three basic reasons to understand stress in the context of engineering rock mechanics:

- There is a pre-existing stress state in the ground and we need to understand it, both directly and as the stress state applies to analysis and design.
- During rock excavation, the stress state can change dramatically. This is because rock, which previously contained stresses, has been removed and the loads must be redistributed.
- Stress is not familiar: it is a tensor quantity and tensors are not encountered in everyday life.

Why Determine *In Situ* Stress?

The basic motivations for *in situ* stress determination are two-fold:

- Engineering analyses require boundary conditions. One of the most important boundary conditions for the analysis of underground excavations is *in-situ* stress.



$$\text{FoS} = \frac{\text{Strength}}{\text{Stress}}$$

- To have a basic knowledge of the stress state (e.g. the direction and magnitude of the major principal stress; the direction in which the rock is most likely to fail; etc.).

Civil and mining engineering
- Stability of underground excavations
- Drilling and blasting
- Pillar design
- Design of support systems
- Prediction of rock bursts
- Fluid flow and contaminant transport
- Dams
- Slope stability

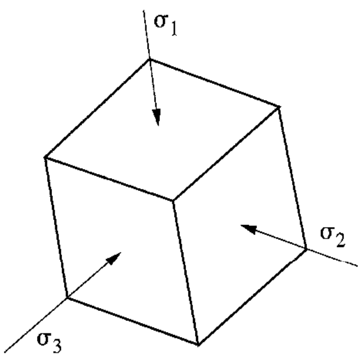
Geology/geophysics
- Earthquake prediction
- Plate tectonics
- Neotectonics
- Structural geology

Energy development
- Borehole stability and deviation
- Fracturing and fracture propagation
- Fluid flow and geothermal problems
- Reservoir production management
- Energy extraction and storage

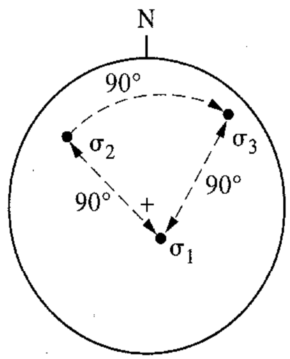
Presentation of In Situ Stress Data

The stress state at a point in a rock mass is generally presented in terms of the magnitude and orientation of the principal stresses (remember that the stress state is completely described by six parameters).

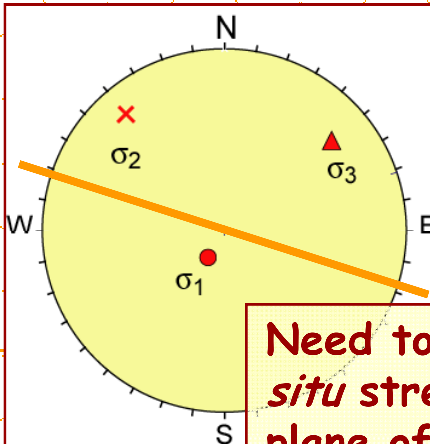
Hudson & Harrison (1997)



$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$



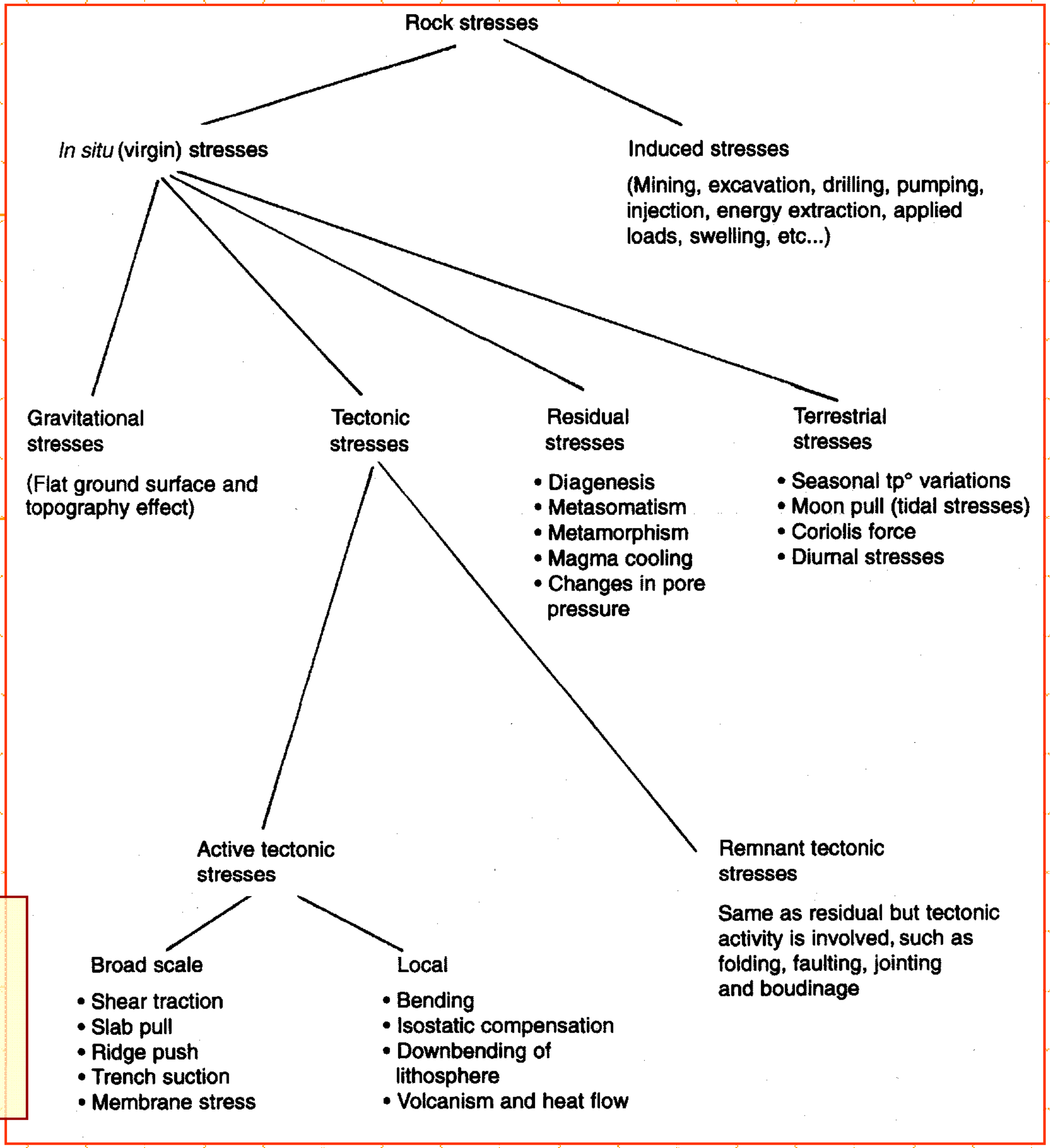
Stress	(MPa)	Trend (°)	Plunge (°)
Sigma 1	10	210	70
Sigma 2	8	320	10
Sigma 3	5	50	15



… principal stresses acting on a cube (left), expressed in matrix form (centre), and shown on a hemispherical projection in terms of their orientation.

Need to know the *in-situ* stress in the plane of a tunnel for plane strain analysis.

In Situ Stress

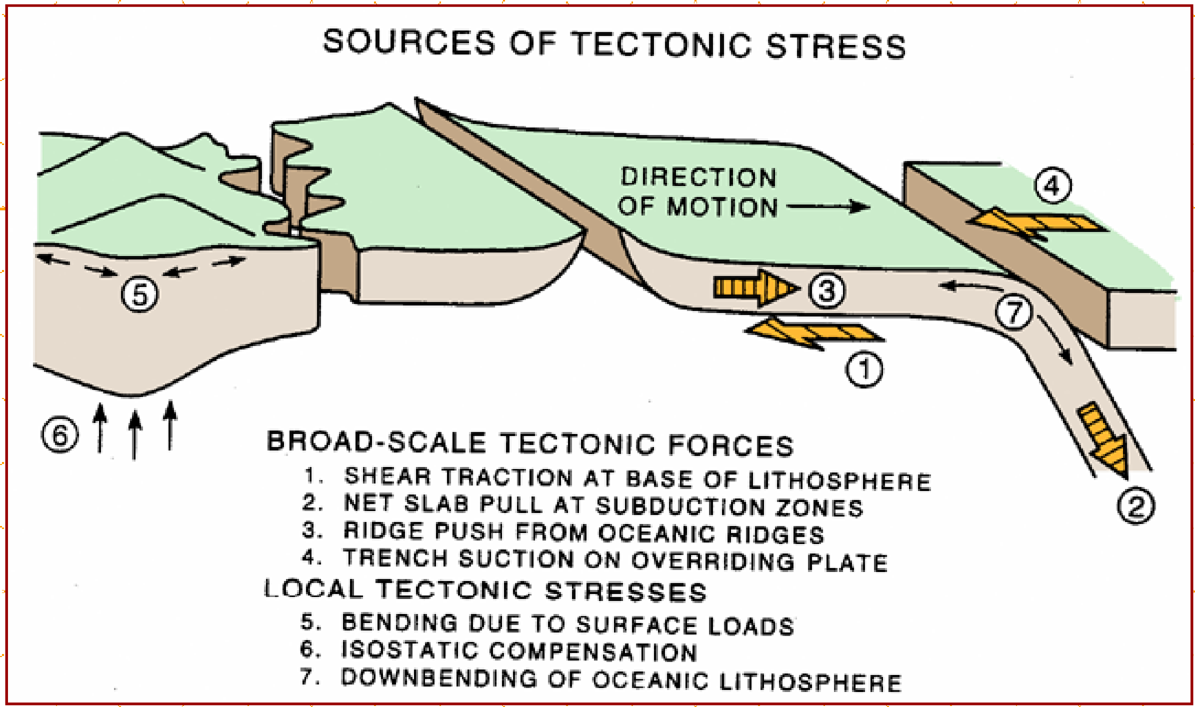


Amadei & Stephansson (1997)

… components of rock stress and stress terminology.

In Situ Stress

When considering the loading conditions imposed on the rock mass, it must be recognized that an *in situ* pre-existing state of stress already exists in the rock.

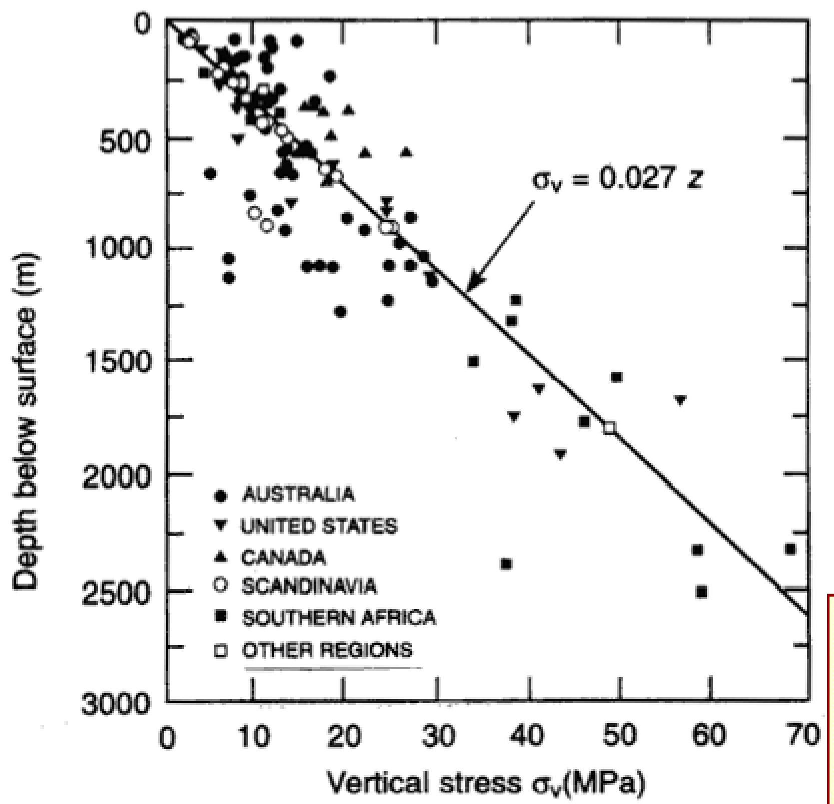


Zoback *et al.* (1989)

… forces responsible for tectonic stresses.

Estimation of *In Situ* Stresses - Vertical

As a first approximation, the principal *in situ* stresses can be assumed to act vertically (one component) and horizontally (two components).



Hoek & Brown (1980)

The vertical stress component is assumed to increase with depth due to the weight of the overburden:

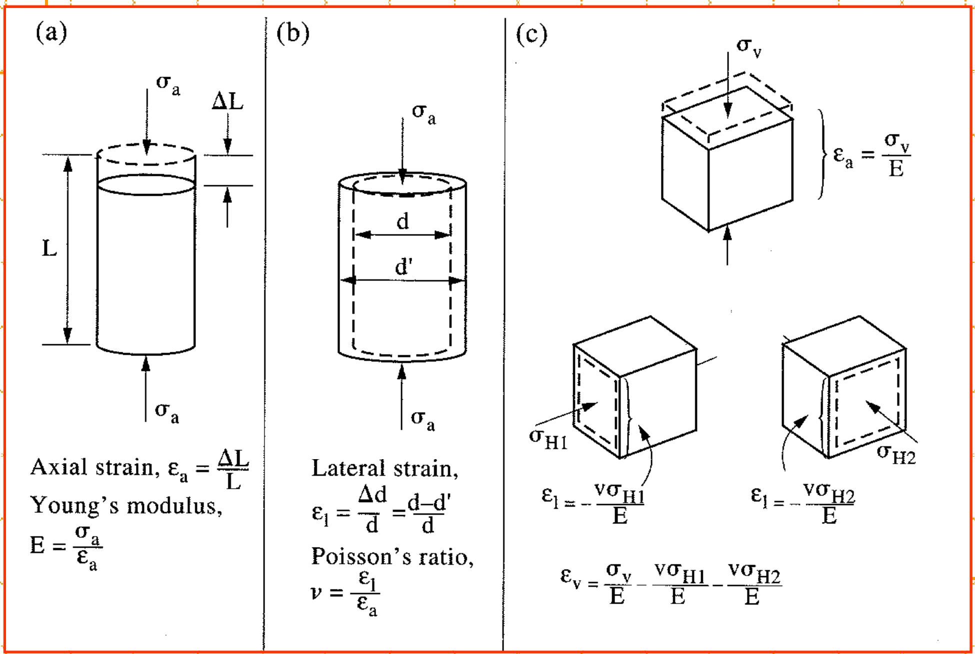
$$\sigma_v = \gamma\ z$$

Where z is the depth, measured in metres below ground surface and γ is the unit weight, measured in MN/m^3.

As a rule of thumb, taking the average density of rock into account, 40 m of overlying rock induces 1 MPa stress.

Estimation of *In Situ* Stresses – Horizontal

The horizontal stress can be estimated using of elastic theory. If we consider the strain along any axis of a small cube at depth, then the total strain can be found from the strain due to the axial stress, subtracting the strain components due to the two perpendicular stresses.



Hudson & Harrison (1997)

For example:

$$\varepsilon_V = \frac{\sigma_V}{E} - \frac{\nu\sigma_{H1}}{E} - \frac{\nu\sigma_{H2}}{E}$$

$$\varepsilon_{H1} = \frac{\sigma_{H1}}{E} - \frac{\nu\sigma_{H2}}{E} - \frac{\nu\sigma_V}{E}$$

Estimation of *In Situ* Stresses - Horizontal

To provide an initial estimate of the horizontal stress, two assumptions are made:

- the two horizontal stresses are equal;
- there is no horizontal strain, i.e. both ε_{H1} and ε_{H2} are zero (e.g. because it is restrained by adjacent elements of rock).

Then we can take ε_{H1} as zero:

$$0 = \frac{\sigma_{H1}}{E} - \frac{\nu\sigma_{H2}}{E} - \frac{\nu\sigma_{V}}{E}$$

And, because $\sigma_{H1} = \sigma_{H2}$:

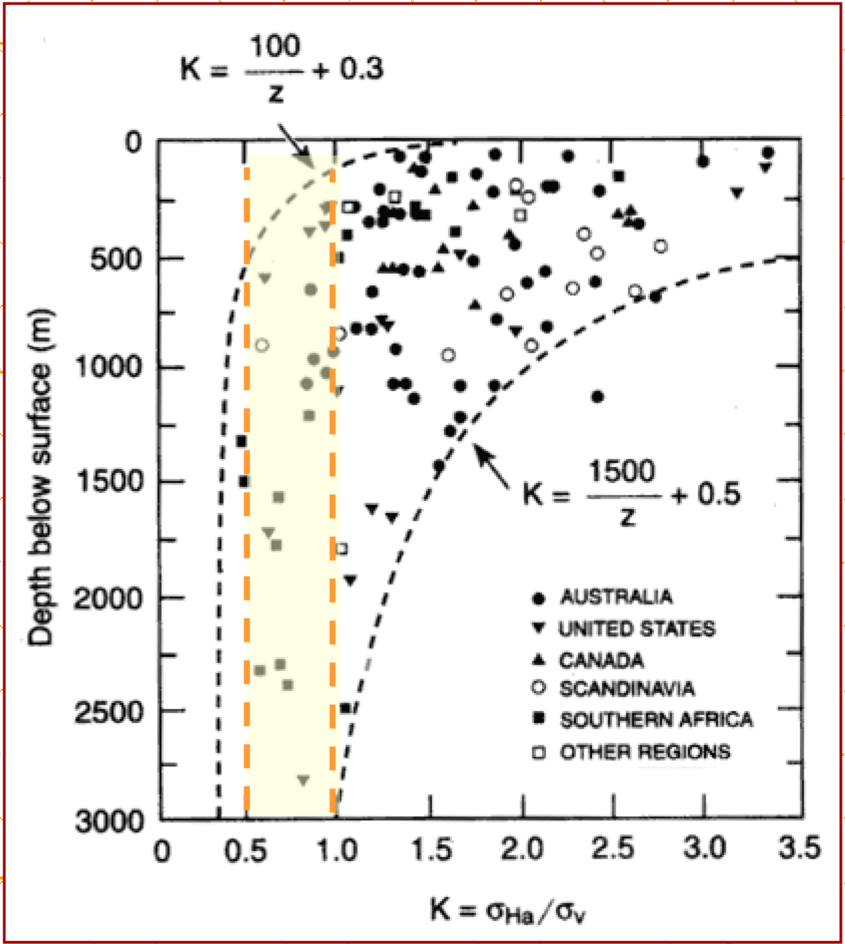
$$\sigma_{H} = \frac{\nu}{1-\nu}\sigma_{V}$$

Estimation of *In Situ* Stresses - Horizontal

Thus the ratio between the horizontal and vertical stress (referred to as $K = \sigma_H/\sigma_V$) is a function of the Poisson's ratio:

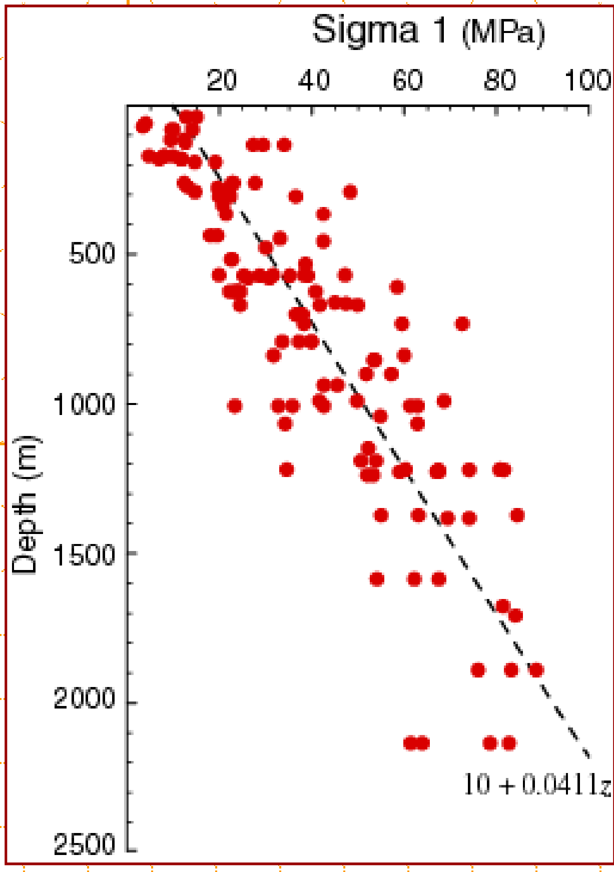
$$\frac{\sigma_H}{\sigma_V} = \frac{\nu}{1-\nu}$$

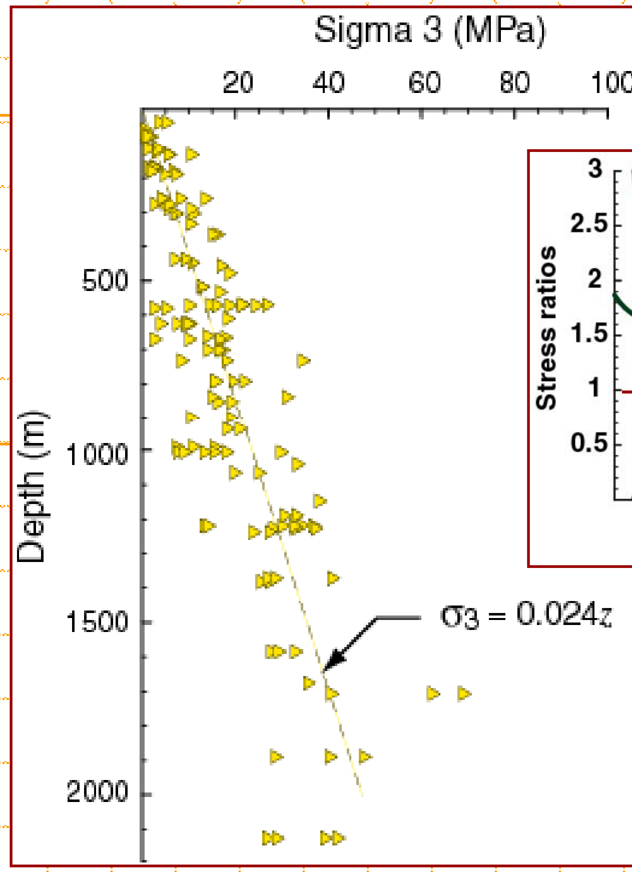
For a typical Poisson's ratio (ν) of 0.25, the resulting K ratio is 0.33. For a theoretical maximum of ν = 0.5, the maximum K ratio predicted is 1.0.

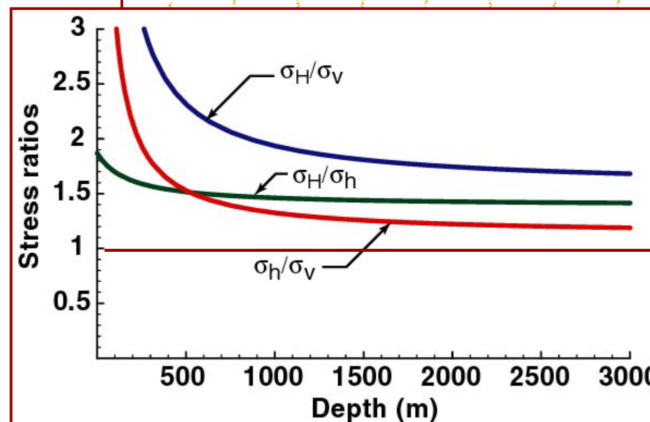


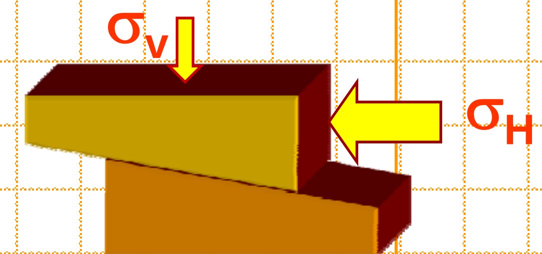
Hoek & Brown (1980)

In Situ Stresses – Canadian Database









Compiled by CANMET: Measurements to 2500 m

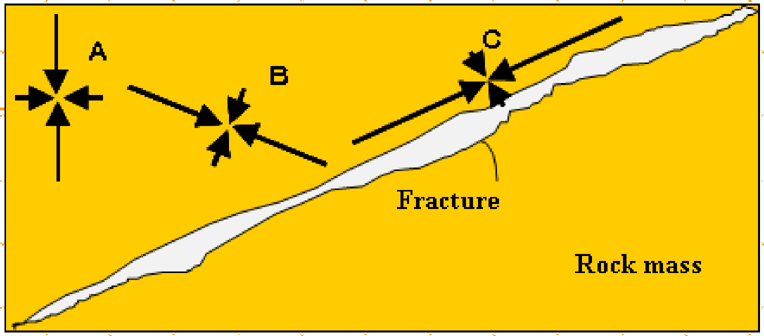
Martin & Chandler (1993)

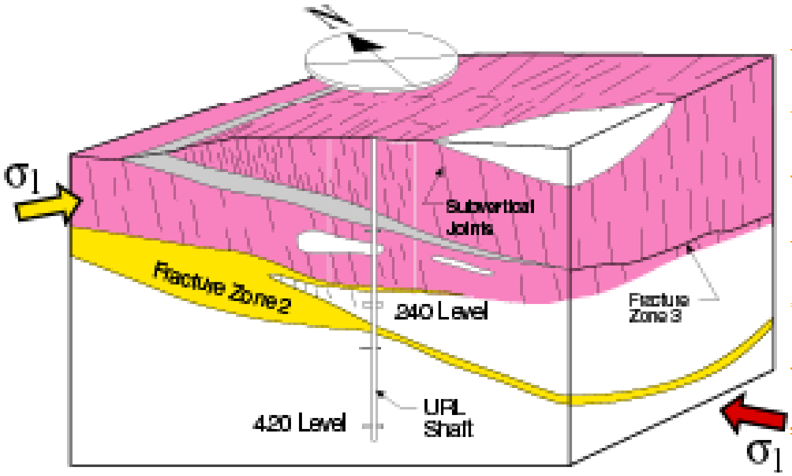
Thrust Faults

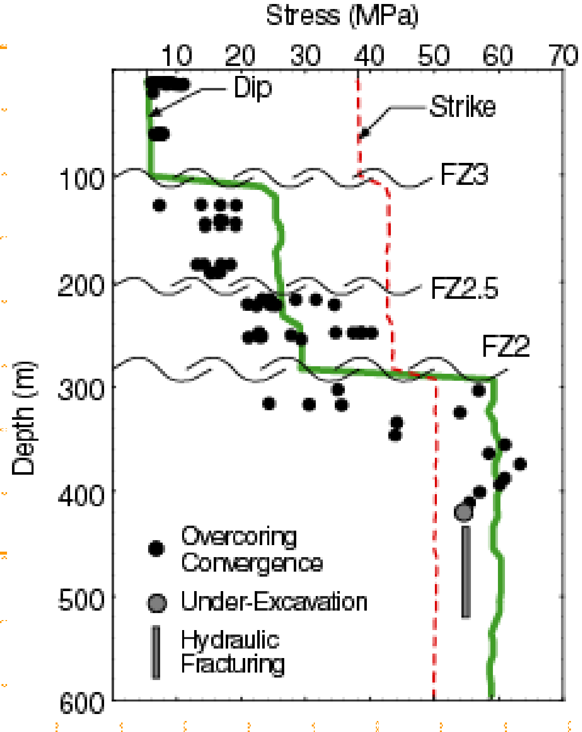
$\sigma_H > \sigma_v$

In Situ Stresses & Geological Structure

Discontinuities, e.g. fault zones, act to dramatically perturb the stress field and thus the magnitudes and orientations of the principal stresses. This may lead to bias if the stress measurements are made near an isolated fracture.







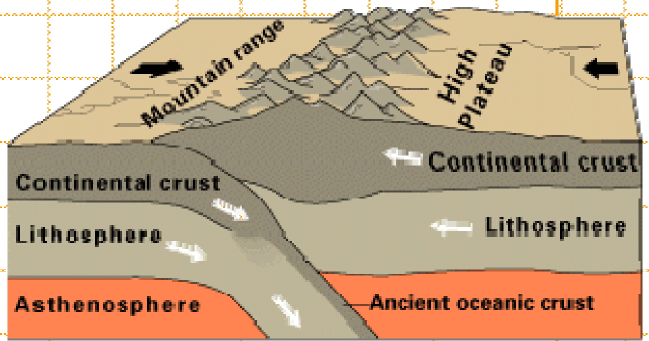
Martin & Chandler (1993)

Reasons for High Horizontal Stresses

High horizontal stresses are caused by factors relating to erosion, tectonics, rock anisotropy, local effects near discontinuities, and scale effects:

Erosion – if horizontal stresses become 'locked in', then the erosion/removal of overburden (i.e. decrease in σ_V) will result in an increase in *K* ratio (σ_H/σ_V).

Tectonics – different forms of tectonic activity (e.g. subduction zones), can produce high horizontal stresses.



Continental-continental convergence

Methods of Stress Determination

Any system utilized for estimating *in situ* stresses should involve a minimum of six independent measurements. Accordingly, there are methods of 'direct' stress measurement and there are methods of estimating the stresses via 'indirect' or 'indicator' methods.

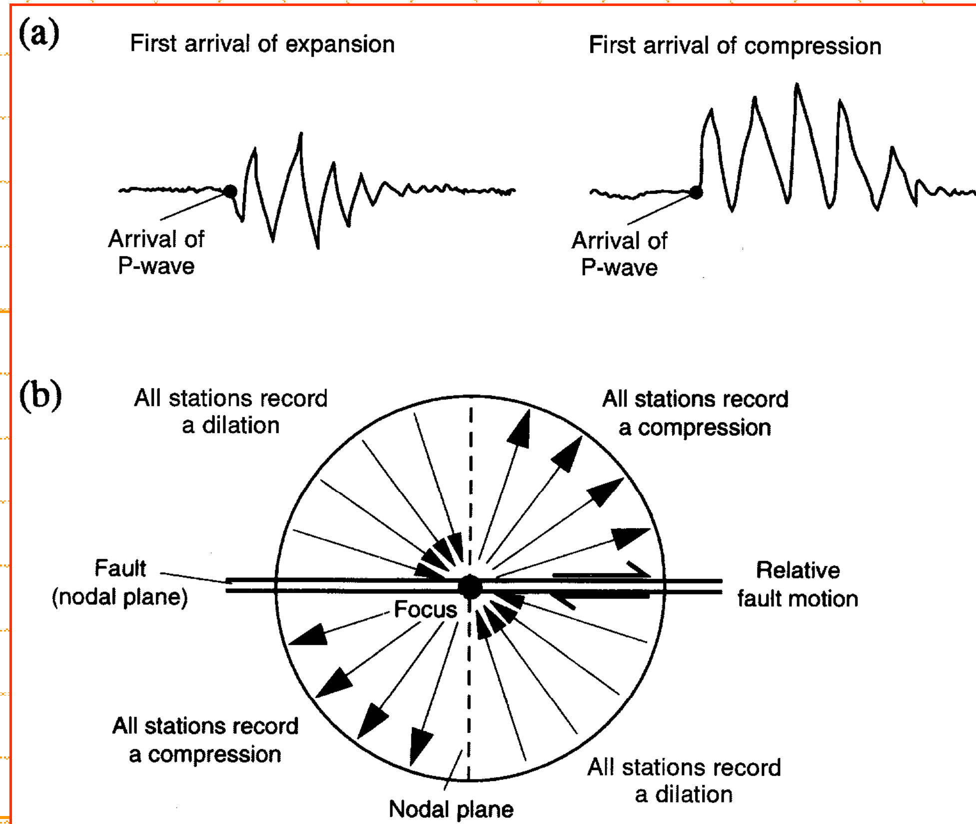
	Method	*Volume (m^3)*
Hydraulic methods	Hydraulic fracturing	0.5–50
	Sleeve fracturing	10^{-2}
	Hydraulic tests on pre-existing fractures (HTPF)	1–10
Relief methods	Surface relief methods	1–2
	Undercoring	10^{-3}
	Borehole relief methods (overcoring, borehole slotting, etc.)	10^{-3}–10^{-2}
	Relief of large rock volumes (bored raise, under-excavation technique, etc.)	10^{2}–10^{3}
Jacking methods	Flat jack method	0.5–2
	Curved jack method	10^{-2}
Strain recovery methods	Anelastic strain recovery (ASR)	10^{-3}
	Differential strain curve analysis (DSCA)	10^{-4}
Borehole breakout method	Caliper and dipmeter analysis	10^{-2}–10^{2}
	Borehole televiewer analysis	10^{-2}–10^{2}
Other methods	Fault slip data analysis	10^{8}
	Earthquake focal mechanisms	10^{9}
	Indirect methods (Kaiser effect, etc.)	10^{-4}–10^{-3}
	Inclusions in time-dependent rock	10^{-2}–1
	Measurement of residual stresses	10^{-5}–10^{-3}

Amadei & Stephansson (1997)

Indicator Methods of Stress Determination

By careful study of earthquake waves recorded by seismographs, it is possible to tell the direction of motion of the fault that caused the earthquake.

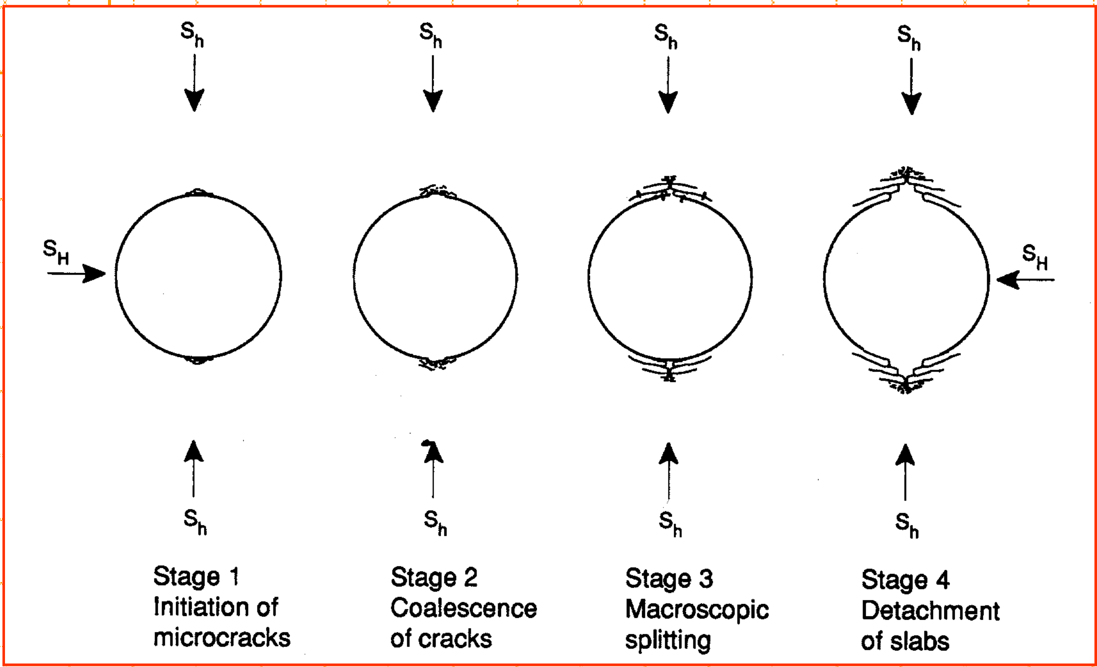
By analyzing the earthquake fault-plane solution (i.e. focal mechanism), a best fit regional stress tensor can be determined by means of an inversion technique.

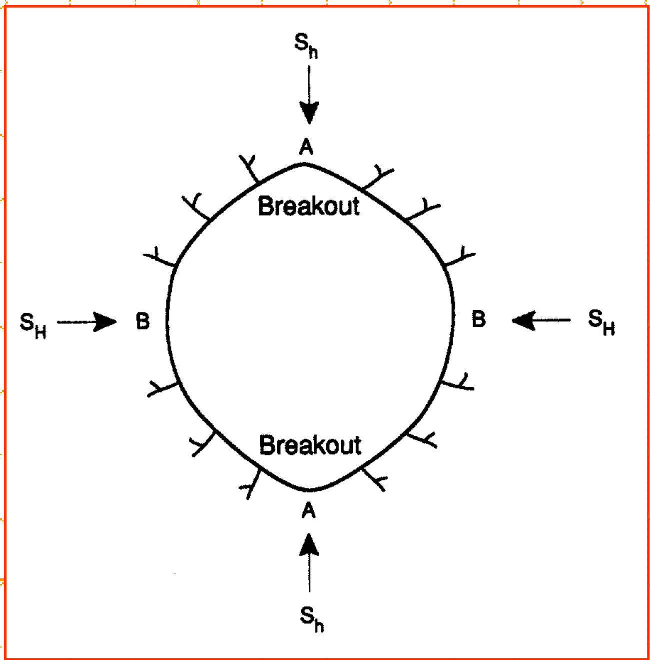


Amadei & Stephansson (1997)

Indicator Methods of Stress Determination

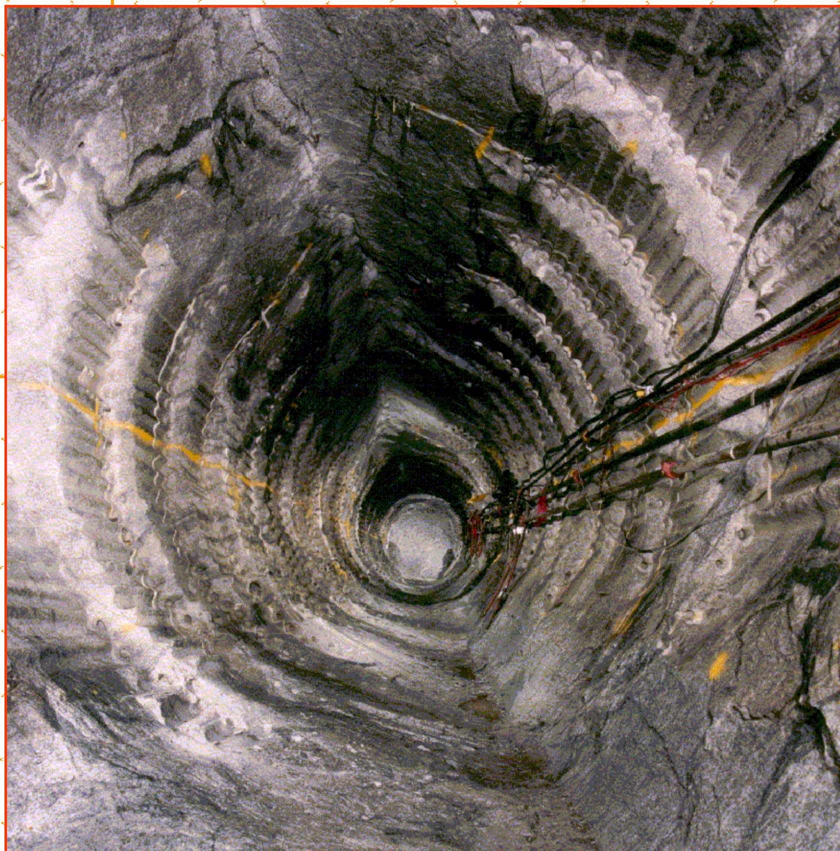
The rock around a circular excavation may not be able to sustain the compressive stress concentration induced during excavation. Failure of the rock results in zones of enlargement called 'breakouts'. There is experimental evidence that breakouts occur in the direction parallel to the minimum *in situ* stress component.

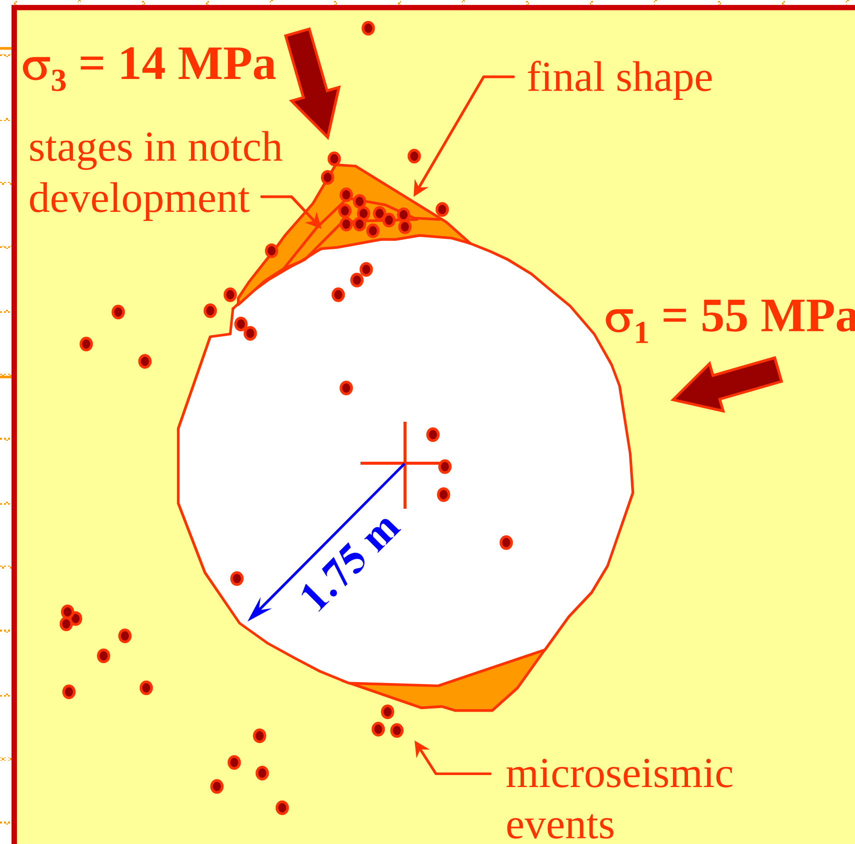




Amadei & Stephansson (1997)

Indicator Methods of Stress Determination

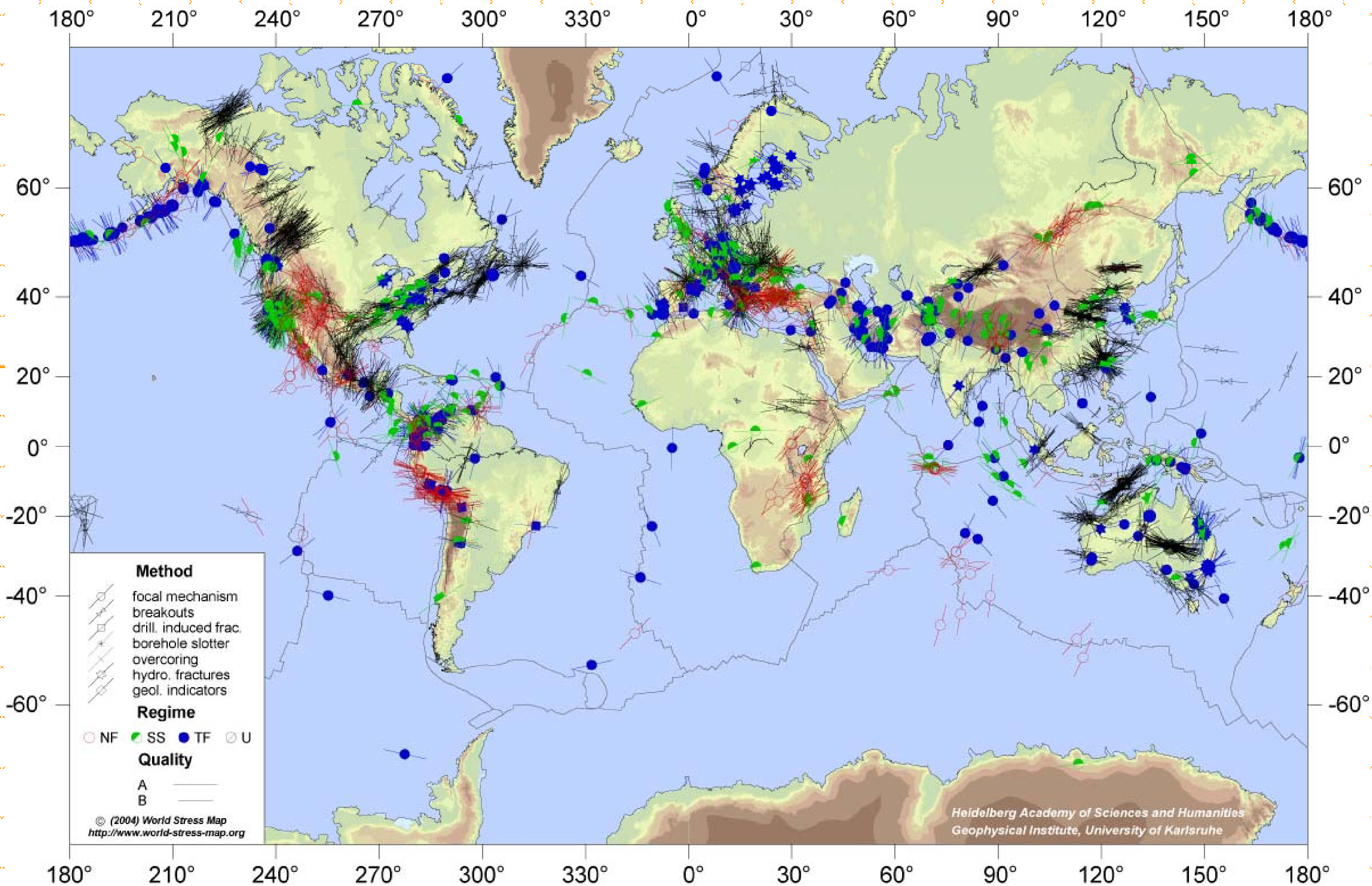




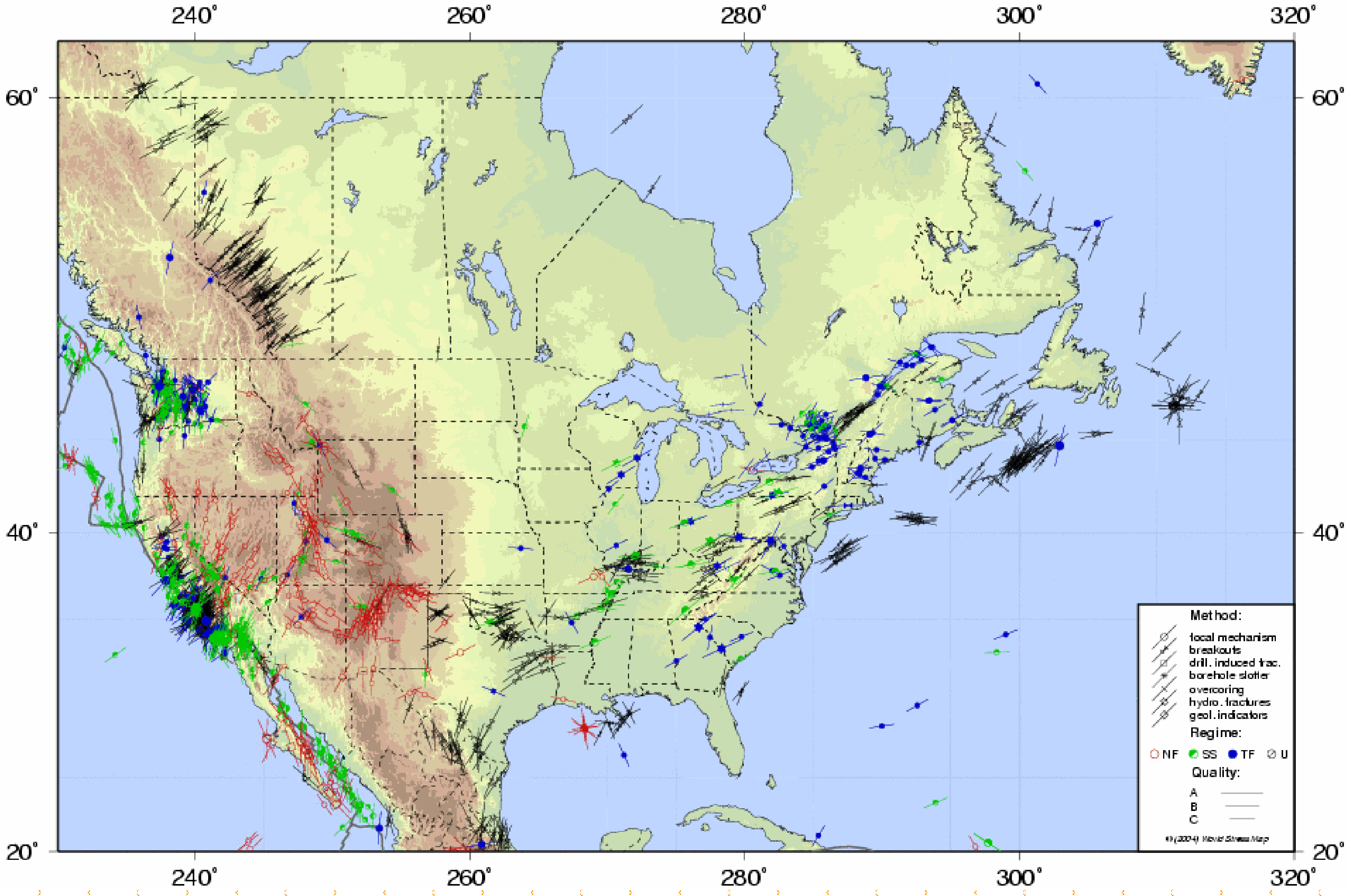
Martin *et al.* (1997)

... stress-controlled tunnel breakout at the URL in Canada.

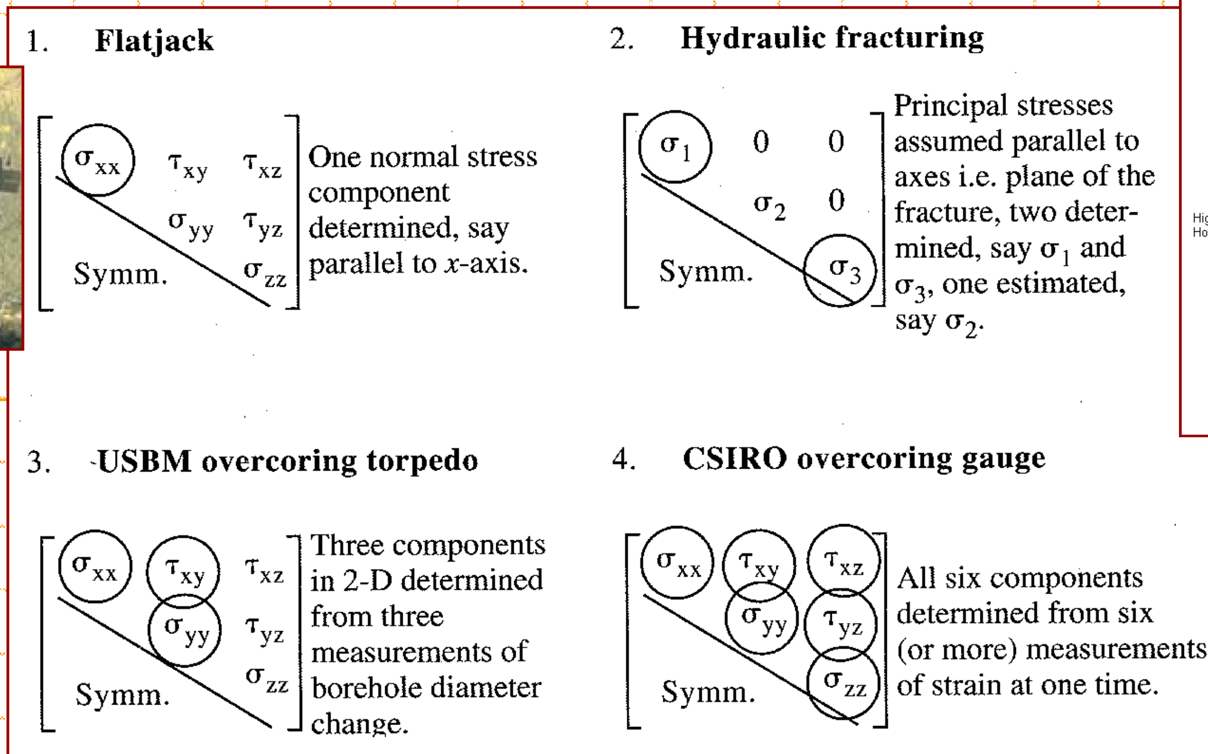
World Stress Map

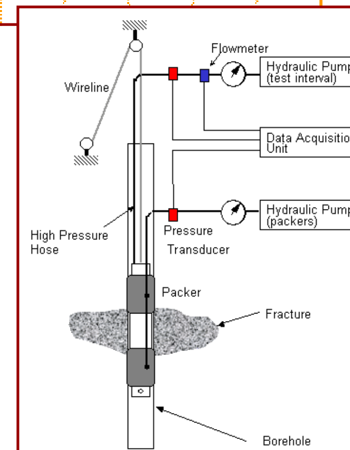


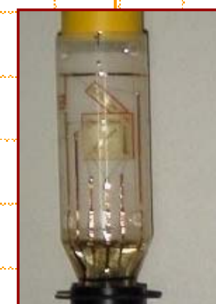
World Stress Map - North America



Methods of Stress Determination





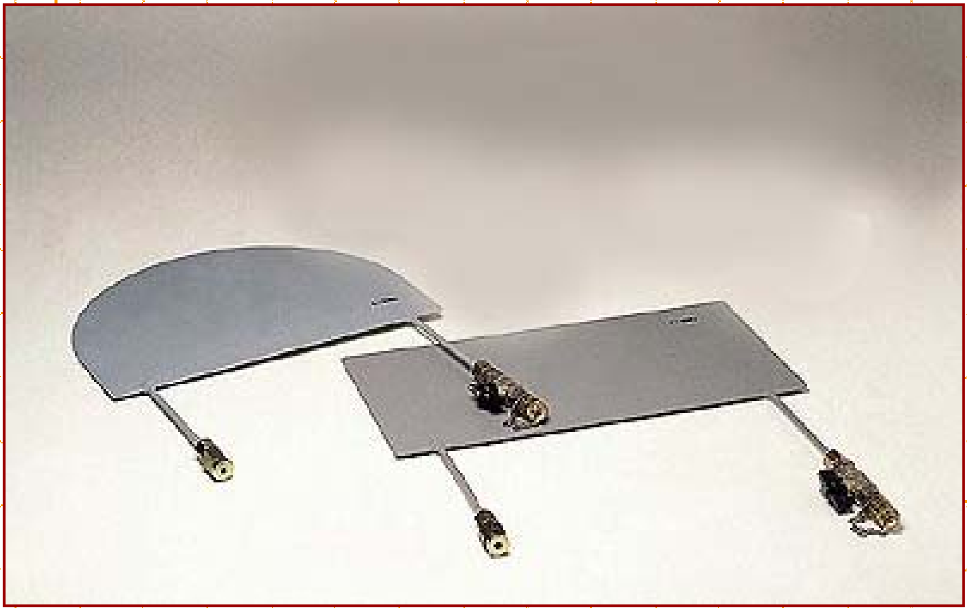


Hudson & Harrison (1997)

… the four ISRM suggested methods for rock stress determination and their ability to determine the 6 independent components of the stress tensor over one test/application of the particular method.

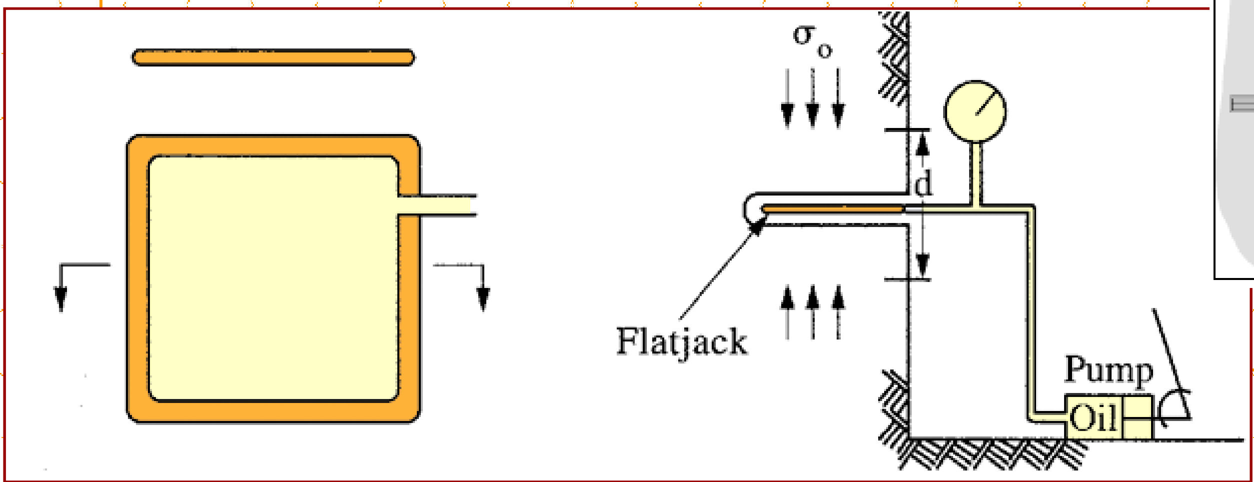
Flatjack Method

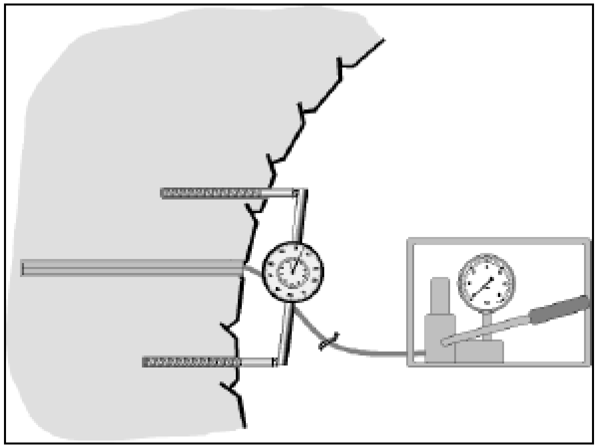
A flatjack is comprised of two metal sheets placed together and welded around their periphery. A feeder tube inserted in the middle allows the flatjack to be pressurized with oil or water.



Flatjack Method

The flatjack method involves the placement of two pins fixed into the wall of an excavation. The distance, *d*, is then measured accurately. A slot is cut into the rock between the pins. If the normal stress is compressive, the pins will move together as the slot is cut. The flatjack is then placed and grouted into the slot.

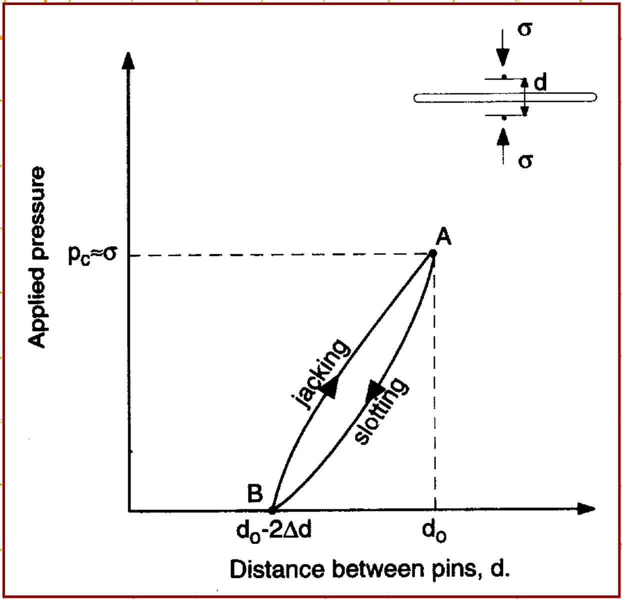


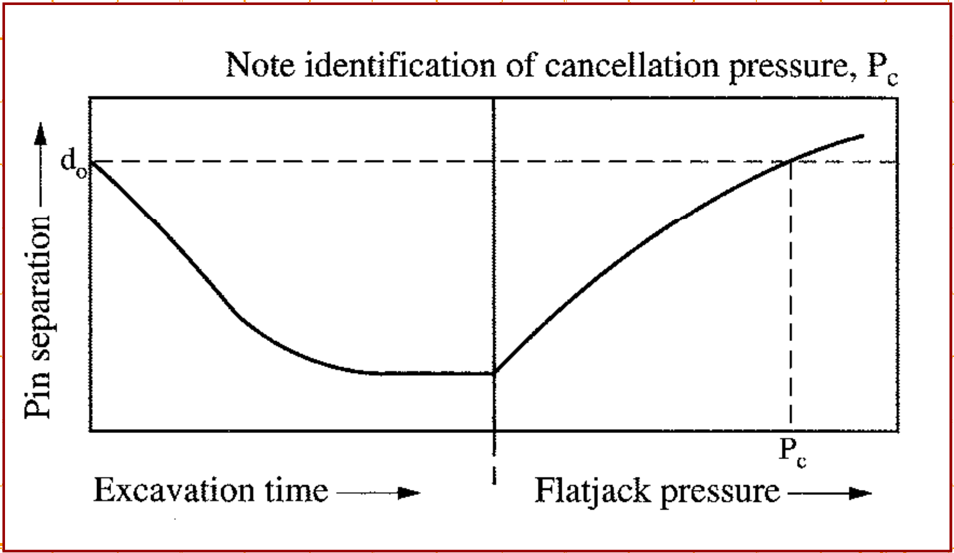


Hudson & Harrison (1997)

Flatjack Method

On pressurizing the flatjack, the pins will move apart. It is assumed that, when the pin separation distance reaches the value it had before the slot was cut, the force exerted by the flatjack on the walls of the slot is the same as that exerted by the pre-existing normal stress.

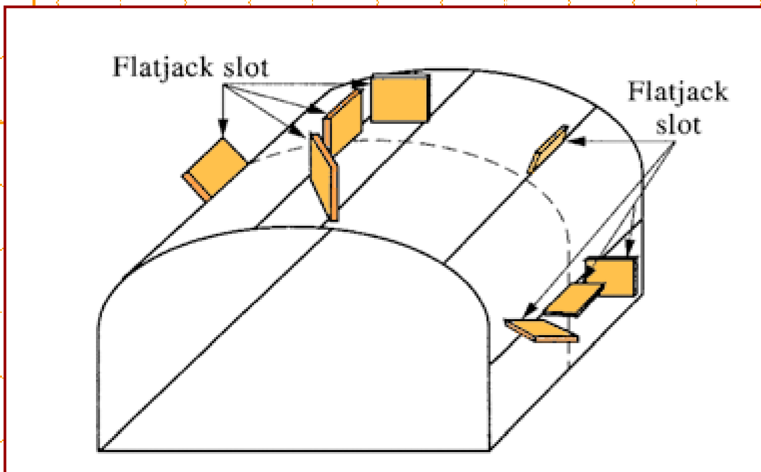


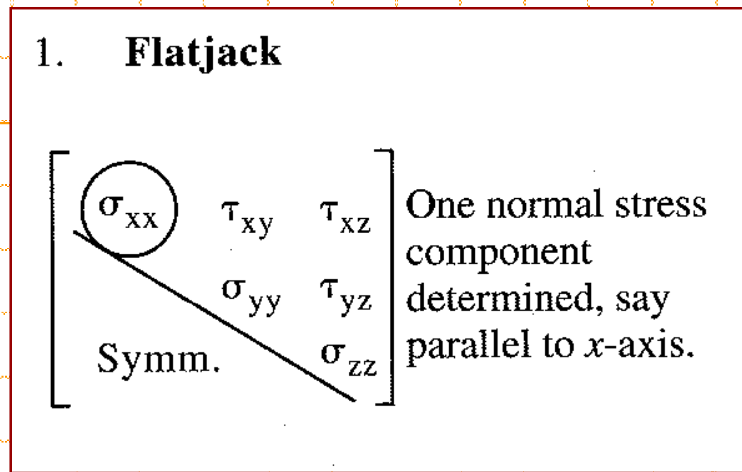


Hudson & Harrison (1997)

Flatjack Method

The major disadvantage with the system is that the necessary **minimum number of 6 tests**, at different orientations, have to be conducted **at 6 different locations** and it is therefore necessary to distribute these around the boundary walls of an excavation.



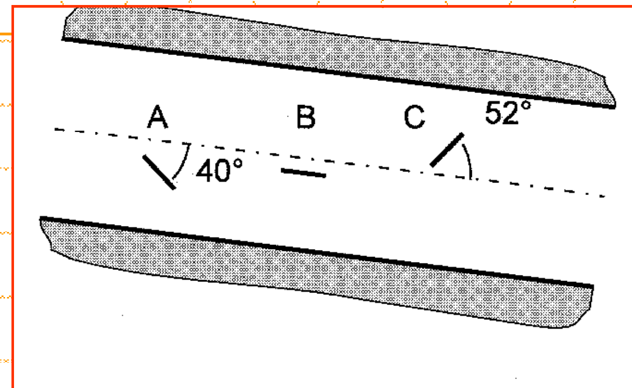


Hudson & Harrison (1997)

It is also important to note that the excavation from which the tests are made will disturb the pre-existing stress state, and so the new redistribution of stresses should be accounted for.

Flatjack Method – Example Problem

Q. Three flatjack tests have been made along a tunnel wall, the axis of which dips at 7°. The measurement position is approximately 250 m below ground surface. The slots for the flatjacks were cut normal to the wall as shown. The cancellation pressures for each flatjack were: A = 7.56 MPa; B = 6.72 MPa; C = 7.50 MPa. Compute the principal stresses and their directions.



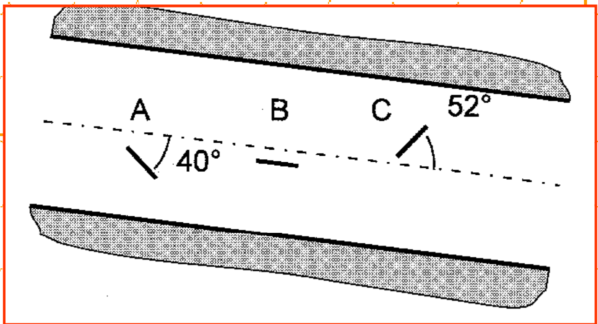
Harrison & Hudson (2000)

A. One way of solving this problem is to use the stress transformation equations, i.e.:

$$\sigma_x' = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cos\theta$$

Flatjack Method – Worked Example

① Taking the *x*-axis horizontal directed to the right, and the *y*-axis vertical upwards, and all measurements measured anticlockwise positive from the positive *x*-axis, we have the following dip angles:



Harrison & Hudson (2000)

$$\beta_{tunnel} = -7^o; \quad \beta_A = -40^o + \beta_{tunnel} = -47^o;$$

$$\beta_B = 0^o + \beta_{tunnel} = -7^o; \quad \beta_C = 52^o + \beta_{tunnel} = 45^o.$$

② Because each flatjack measures the normal stress component perpendicular to it, we add 90° to each of these directions to obtain the direction of the normal stress on each flatjack:

Jack A	$\sigma_A = 7.56$ MPa;	$\theta_A = \beta_A + 90^o$;	$\theta_A = 43^o$
Jack B	$\sigma_B = 6.72$ MPa;	$\theta_B = \beta_B + 90^o$;	$\theta_B = 83^o$
Jack C	$\sigma_C = 7.50$ MPa;	$\theta_C = \beta_C + 90^o$;	$\theta_C = 135^o$

③ Assembling the stress transformation equation for all three flatjacks into matrix form gives:

$$\begin{bmatrix} \sigma_A \\ \sigma_B \\ \sigma_C \end{bmatrix} = \begin{bmatrix} \cos^2\theta_A & \sin^2\theta_A & 2\sin\theta_A\cos\theta_A \\ \cos^2\theta_B & \sin^2\theta_B & 2\sin\theta_B\cos\theta_B \\ \cos^2\theta_C & \sin^2\theta_C & 2\sin\theta_C\cos\theta_C \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} \quad \text{or } \boldsymbol{\sigma}_{jack} = \mathbf{R}\boldsymbol{\sigma}_{global}$$

Flatjack Method – Worked Example

④ Evaluation of this matrix gives:

$$\begin{bmatrix} 7.56 \\ 6.72 \\ 7.50 \end{bmatrix} = \begin{bmatrix} 0.535 & 0.465 & 0.998 \\ 0.015 & 0.985 & 0.242 \\ 0.500 & 0.500 & -1.000 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Which upon inversion gives $\sigma_{global} = R^{-1}\sigma_{jack}$, or:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} 1.093 & -0.952 & 0.860 \\ -0.134 & 1.021 & 0.113 \\ 0.479 & 0.034 & -0.514 \end{bmatrix} \begin{bmatrix} 7.56 \\ 6.72 \\ 7.50 \end{bmatrix}$$

⑤ From this we find:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} 8.31 \\ 6.70 \\ 0.00 \end{bmatrix} \text{MPa.}$$

We see that σ_x and σ_y are principal stresses, because $\tau_{xy} = 0$, and the principal stresses are vertical and horizontal, which is a reasonable finding.

Note that the horizontal stress, σ_x, is greater than the vertical stress, σ_y, which is a common occurrence. Now if we were to compare the weight of the overburden to the computed vertical stress value:

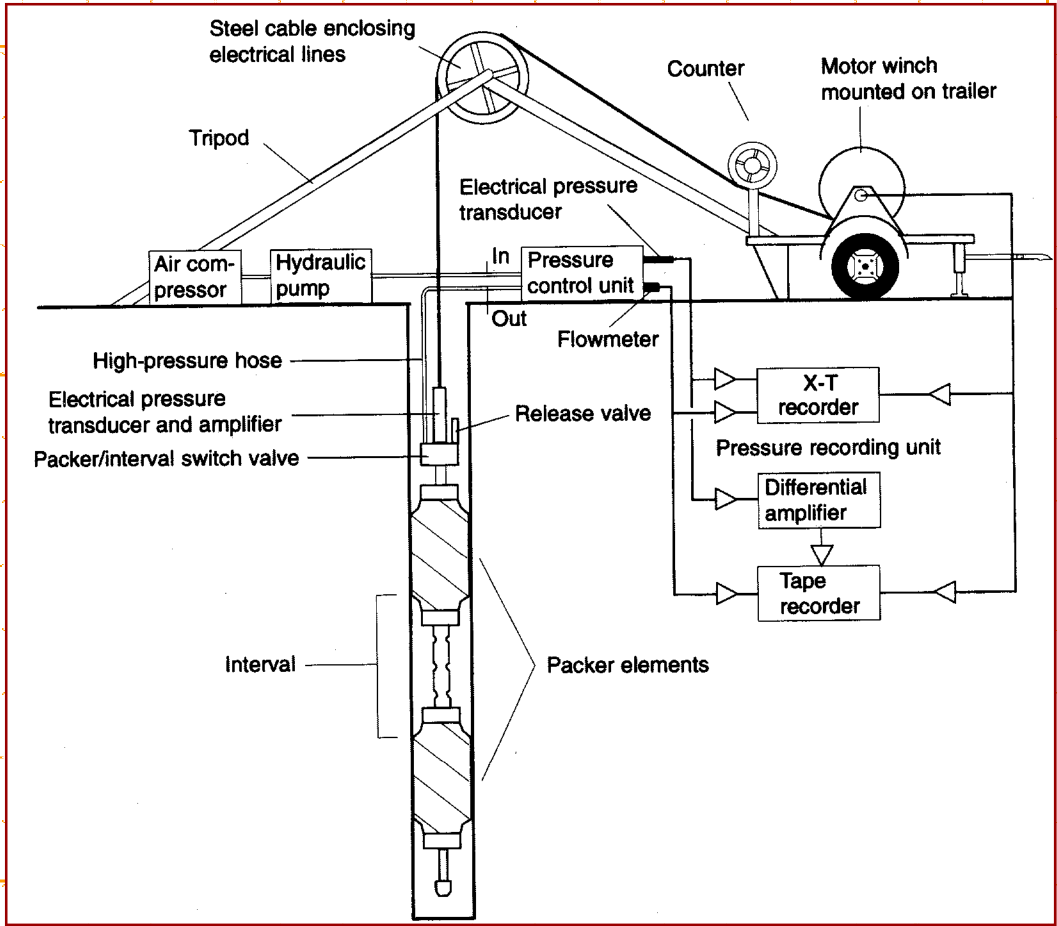
$$\gamma_{rock} = 27 \text{ kN/m}^3; \quad z = 250 \text{ m}; \quad \sigma_v = \gamma_{rock} \times z; \quad \sigma_v = 6.75 \text{ MPa.}$$

This compares well with the value found for σ_y.

Harrison & Hudson (2000)

Hydraulic Fracturing Method

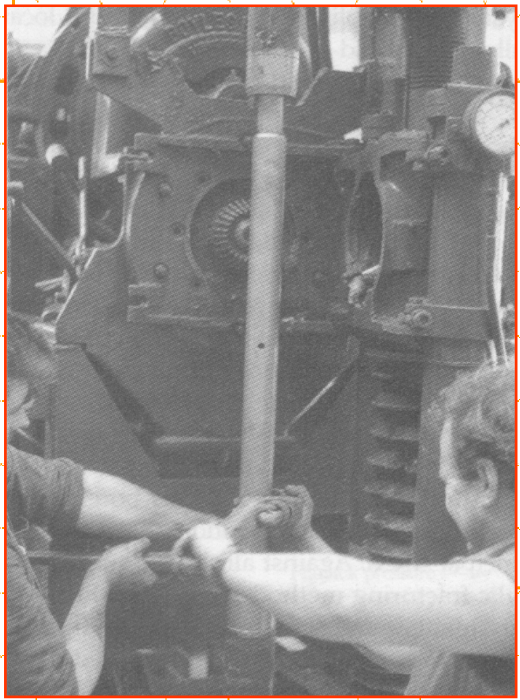
The **hydraulic fracturing method** involves the pressuring of a borehole interval, typically 1 m long, isolated using a straddle packer system. The isolated zone is **pressurized** by water until a fracture occurs in the rock.

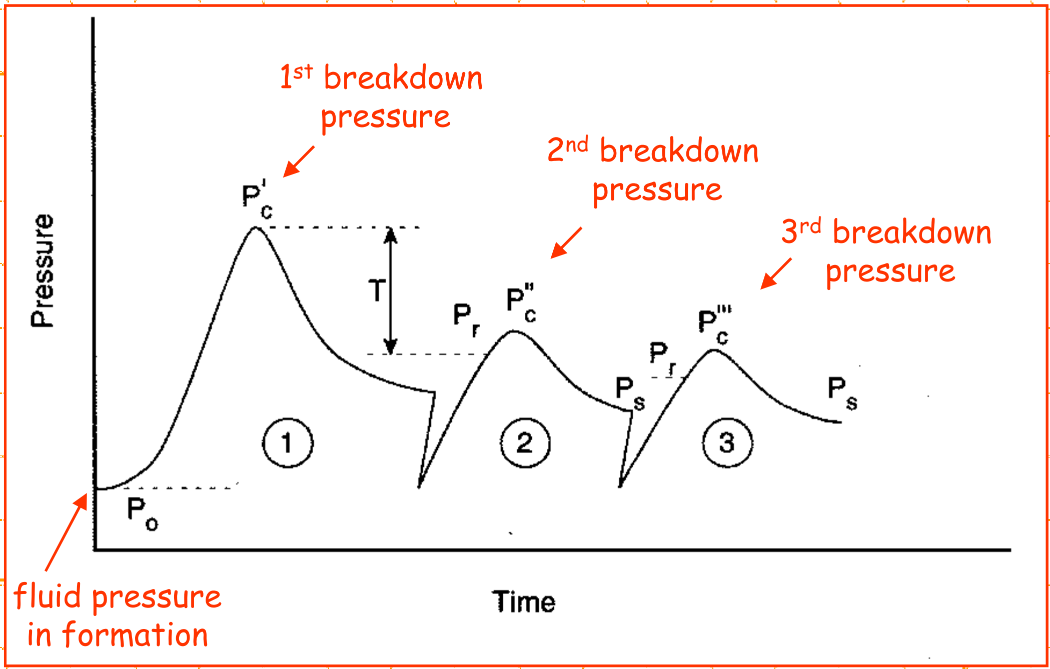


Amadei & Stephansson (1997)

Hydraulic Fracturing Method

The two measurements taken are the water pressure when the fracture occurs and the subsequent pressure required to hold the fracture open. These are referred to as the breakdown pressure (P_c' or P_B) and the shut-in pressure (P_s).



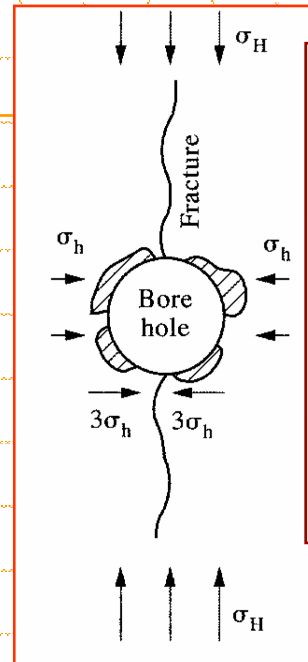


Amadei & Stephansson (1997)

Hydraulic Fracturing Method

In calculating the *in* situ stresses, the shut-in pressure (P_s) is assumed to be equal to the minor horizontal stress, σ_h.

The major horizontal stress, σ_H, is then found from the breakdown pressure (P_c' or P_B). In this calculation, the breakdown pressure has to overcome the minor horizontal principal stress (concentrated three times by the presence of the borehole) and overcome the *in situ* tensile strength of the rock; it is assisted by the tensile component of the major horizontal principal stress.



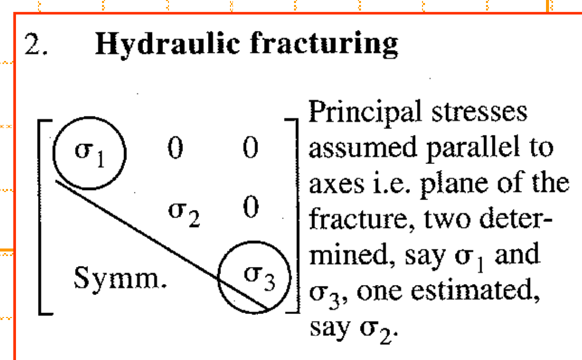
Relationships

$\sigma_h = P_s$

$\sigma_H = 3\sigma_h - P_c' - P_o + \sigma_t$

$\sigma_t = P_c' - P_r$

$\sigma_H = 3\sigma_h - P_r - P_o$



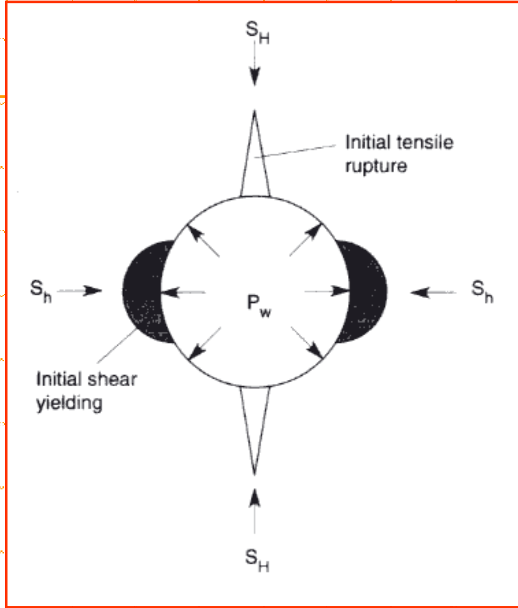
Hudson & Harrison (1997)

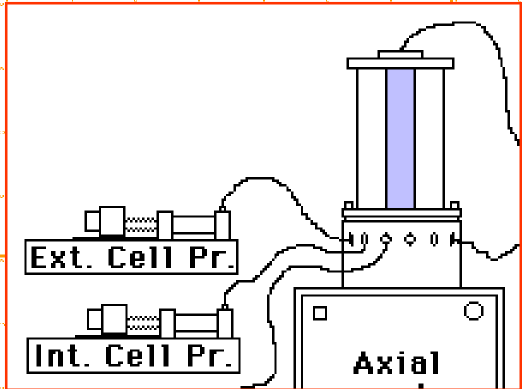
Hydraulic Fracturing Method

The analysis assumes that the induced fracture has propagated in a direction perpendicular to the minor principal stress.

Other assumptions include that of elasticity in the rock forming the borehole wall (from which the borehole stress concentration factor of three is derived), and impermeability of the host rock so that pumped water has not significantly penetrated the rock and affected the stress distribution.

The tensile strength of the rock can be obtained from test performed by pressurizing hollow rock cylinders.

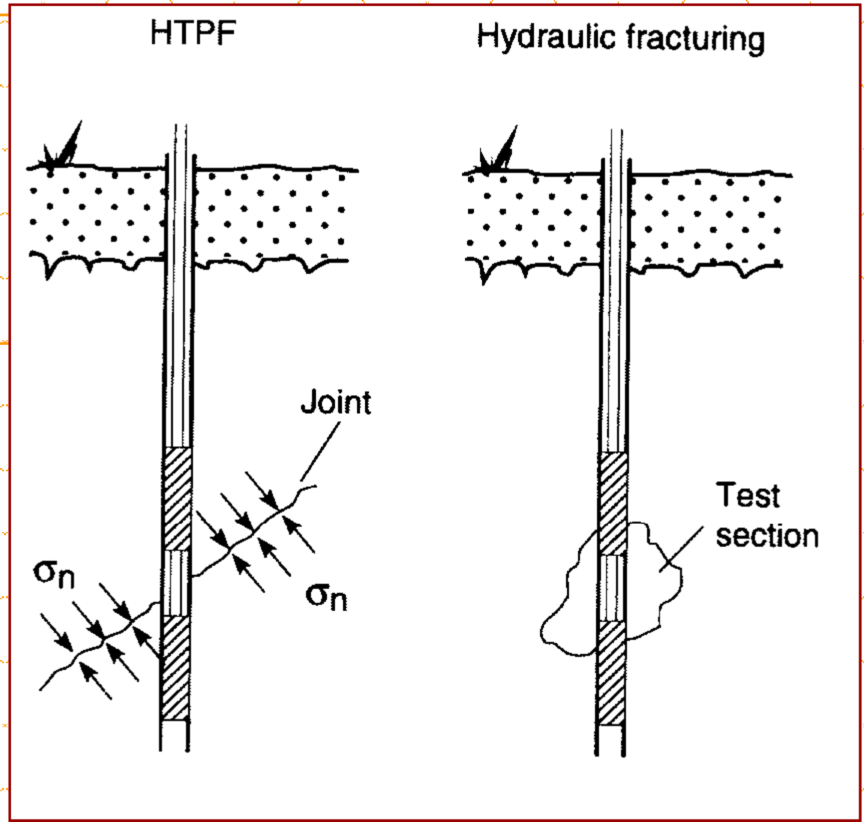




Hydraulic Fracturing Method - HTPF

The HTPF method (Hydraulic Testing on Pre-existing Fractures), consists of reopening an existing fracture of known orientation that has previously been isolated in between two packers. By using a low fluid injection rate, the fluid pressure which balances exactly the normal stress across the fracture is measured.

The method is then repeated for other non-parallel fractures of known orientation.

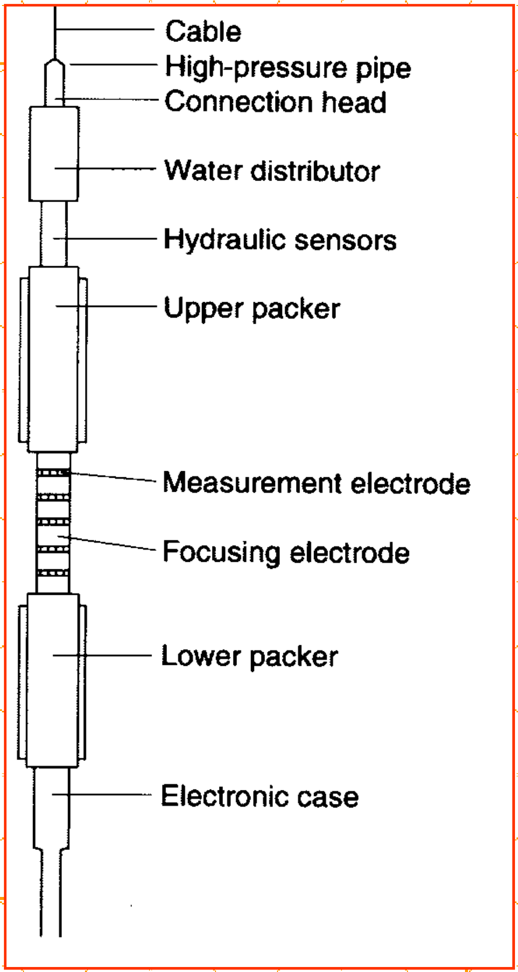


Amadei & Stephansson (1997)

Hydraulic Fracturing Method - HTPF

By determining the normal stresses acting across several non-parallel fractures and knowing their orientation, a system of equations can be created to determine the six *in situ* stress components without making any assumption with regards to the orientation of the principal stresses and the rock's constitutive behaviour.

It is the only hydraulic method that does not have to assume that the principal stress directions are aligned vertically and horizontally.



Cornet (1993)

Hydraulic Fracturing Method – Worked Example

Q. A hydraulic fracture test in a granite rock mass yield the following results:

Depth (m)	Breakdown pressure, P_B (MPa)	Shut-in pressure, P_S (MPa)
500	14.0	8.0

Given that the tensile strength of the rock is 10 MPa, estimate the principal stresses assuming one is vertical and that the pressure values were adjusted to account for the formation pressures (i.e. P_o=0 for calculation purposes).

A. Assuming that the rock mass was behaving as an elastic material...

Relationships

$\sigma_h = P_s$

$\sigma_H = 3\sigma_h - P_c' - P_o + \sigma_t$

1. Calculate the min. horizontal stress: $\sigma_h = P_s = 8$ MPa

2. Calculate the max. horizontal stress: $\sigma_H = 3\sigma_h - P_c' - \cancel{P_o} + \sigma_t$ (P_o = 0)

$$\sigma_H = 3(8\text{ MPa}) - 14\text{ MPa} + 10\text{ MPa} \Rightarrow \sigma_H = 20\text{ MPa}$$

3. The vertical stress can now be estimated from the overburden (assume γ = 27 kN/m³ for granite): $\sigma_v = 500\text{m} * 0.0027\text{ MN/m}^3 = 13.5$ MPa

$\sigma_1 = \sigma_H = 20$ MPa $\sigma_2 = \sigma_v = 13.5$ MPa $\sigma_3 = \sigma_h = 8$ MPa

Borehole Relief Methods - Overcoring

The main idea behind relief methods is to isolate (partially or wholly) a rock sample from the stress field that surrounds it and to monitor the response. As such, the stresses are not related to applied pressures, such as with the hydraulic tests. Instead, the stresses are inferred from strains generated by the relief (unloading) process and measured directly on the rock associated with the relief process.

Overcoring methods are by far the most commonly used relief method.

Surface relief methods	● Isolate a block of rock from surrounding rock mass and monitor its surface strain or deformation response: ● Monitor hole deformation due to drilling of parallel hole ● Center hole drilling or undercoring
Borehole relief methods	● Overcoring of prestressed cells ● Overcoring of deformation-type gages such as the USBM gage ● Overcoring of a gage attached to the flat end of a borehole: Doorstopper and photoelastic disks ● Overcoring of CSIR-type triaxial strain cells ● Overcoring of triaxial strain cells attached to the end of a borehole (spherical and conical cells) ● Overcoring of stiff, solid or hollow inclusion-type gages ● Borehole jack fracturing, or slotting, or deepening ● Holographic methods ● Undercoring of borehole wall ● Borehole tapercoring
Rock mass relief methods	● Bored raise method ● Back-analysis ● Under-excavation technique

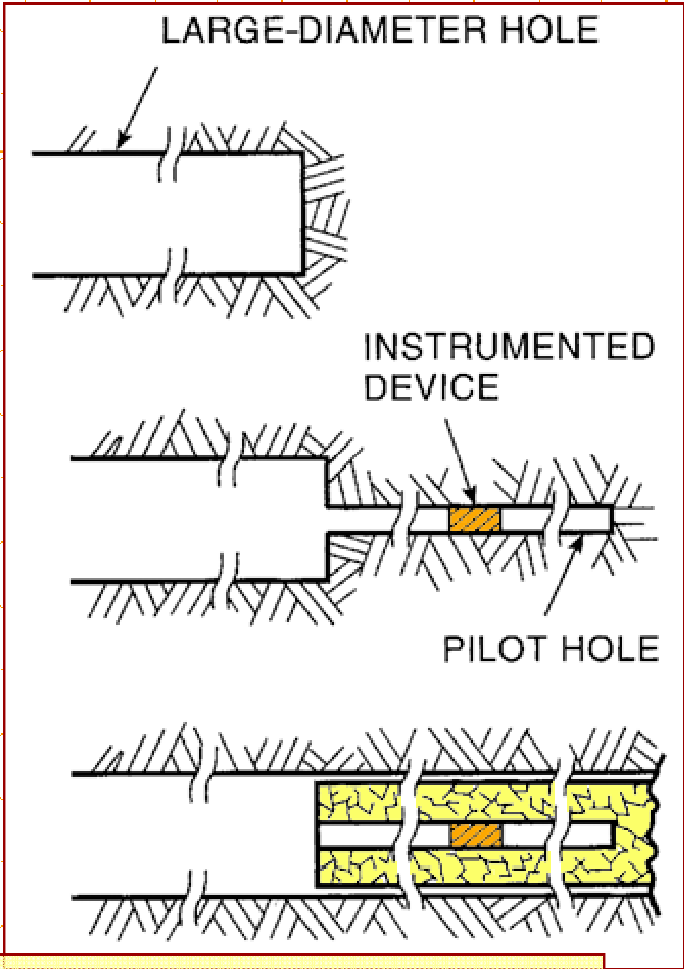
Amadei & Stephansson (1997)

Overcoring Method

First, a large diameter borehole is drilled (between 60 and 220 mm) to a sufficiently large distance so that stress effects due to any excavations can be neglected.

Second, a small pilot hole (e.g. 38 mm) is drilled. The measuring device is then inserted and fastened in this hole.

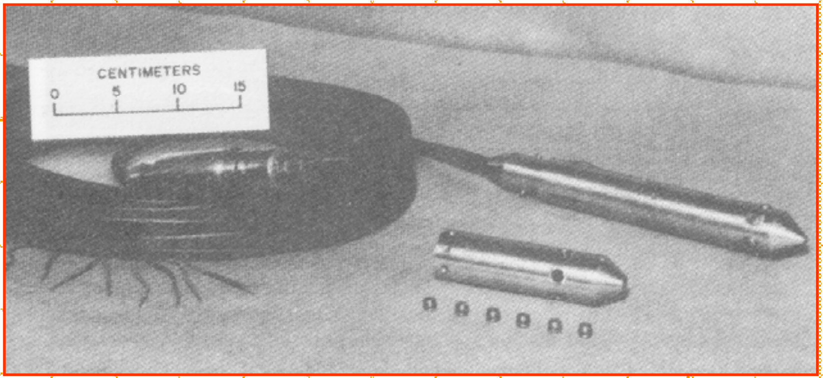
Thirdly, the large diameter hole is resumed, relieving stresses and strains in the hollow rock cylinder that is formed. Changes in strain are then recorded with the instrumented device as the overcoring proceeds past the plane of measurement.



Following overcoring, the recovered overcore (containing the instrumented device) is then tested in a biaxial chamber to determine the elastic properties of the rock.

Overcoring Method – USBM Deformation Probe

The USBM technique (from the U.S. Bureau of Mines) allows the complete stress state to be determined from three measurements in boreholes with different orientations when the stresses are released by overcoring the borehole.

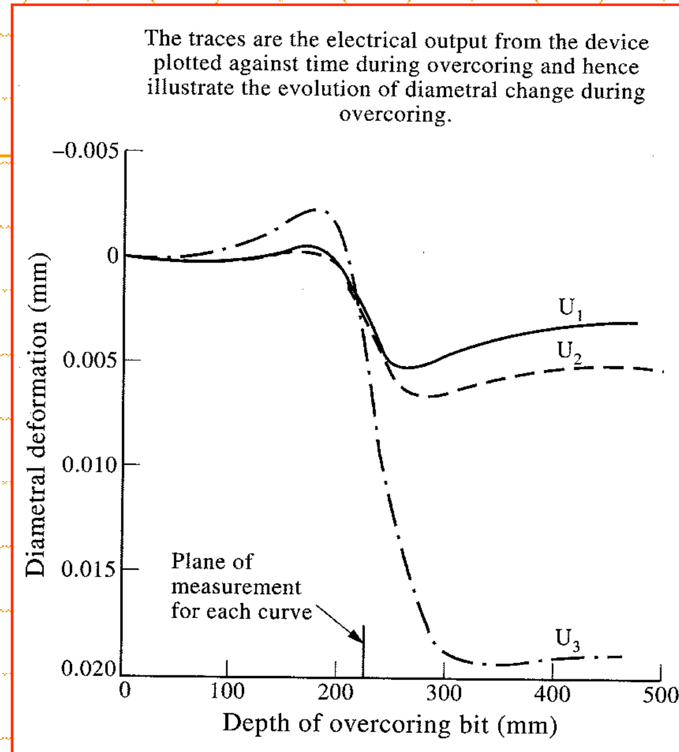


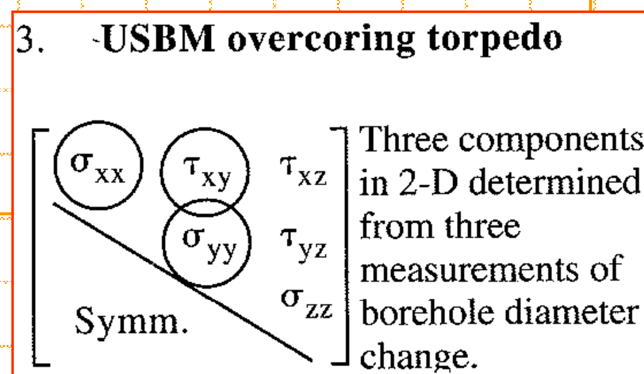
When the probe is inserted in a borehole, six 'buttons' press against the borehole wall and their diametral position is measured by strain gauges bonded to steel cantilevers supporting the buttons.

Overcoring Method – USBM

When the borehole is overcored by a larger diameter borehole, the stress state in the resulting hollow cylinder is **reduced to zero**, the diameter of the hole changes, the buttons move, and hence different **strains are induced** in the strain gauges.

From these changes, and with the use of **elasticity theory**, the biaxial stress state in the plane perpendicular to the borehole axis is deduced.

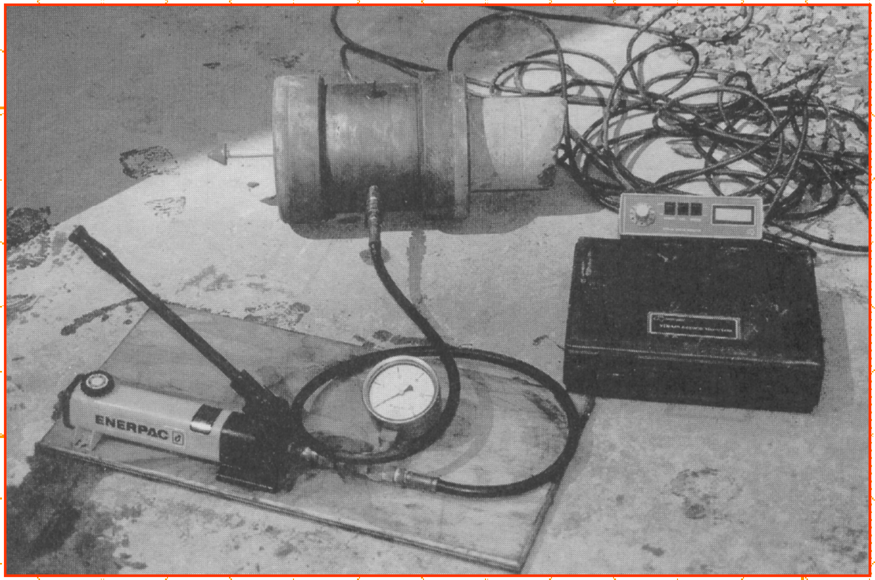




Hudson & Harrison (1997)

Overcoring Method – USBM

A useful aspect of this technique is that it produces an annular core which may be used in the laboratory to determine the elastic properties directly.

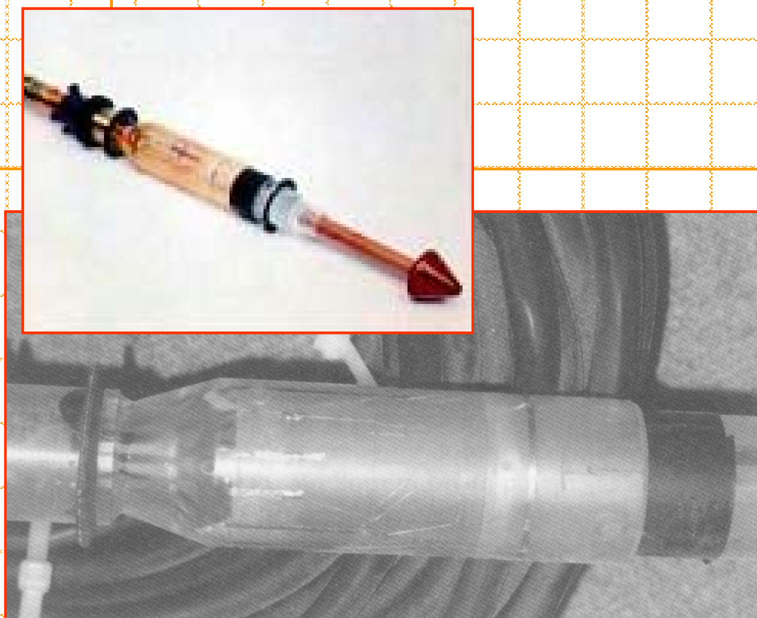


Given the validity of the assumptions, the USBM probe is efficient because it is reusable, permit measurements to be made many times within a borehole and are relatively cheap and robust.

However, the analysis can be complicated by the presence of the borehole, which perturbs the stress state from its natural *in situ* state.

Overcoring Method – CSIRO Hollow Inclusion Cell

The CSIRO device operates on a similar principle to the USBM probe except that it is a gauge which is glued into the borehole and can measure normal strains at a variety of orientations and locations around the borehole wall.

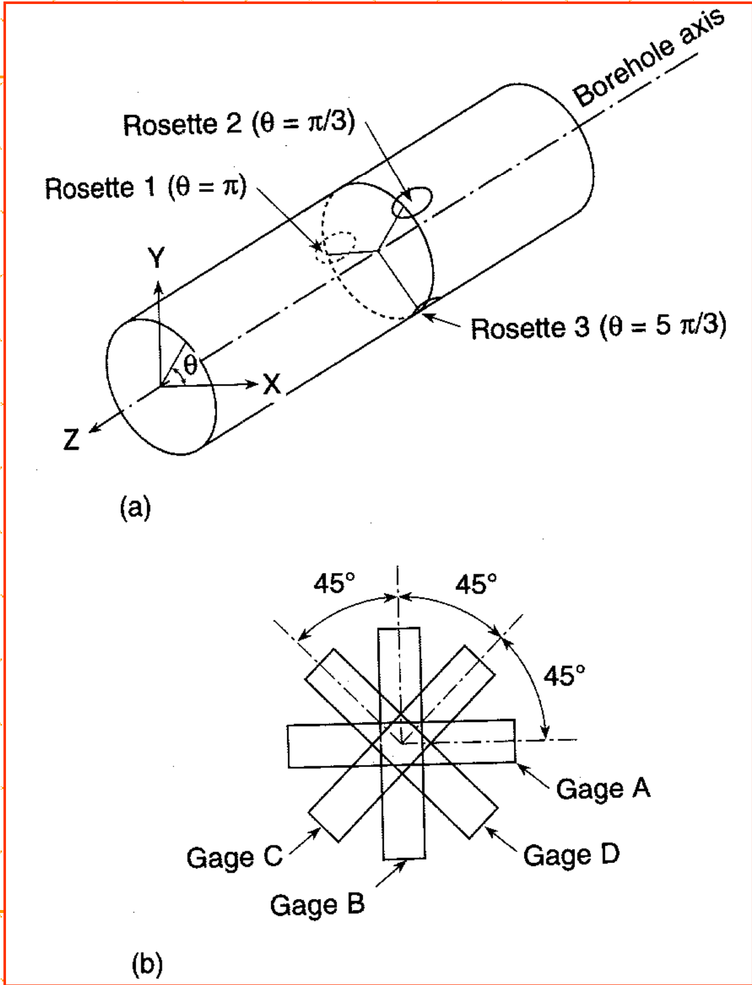


The gauge is glued into position within the pilot hole, initial readings of strain are taken and the gauge is then overcored.

Overcoring Method – CSIRO Hollow Inclusion Cell

Overcoring destresses the resulting hollow cylinder and final strain gauge readings are taken. The gauge has either 9 or 12 separate strain gauges, in rosettes of three, so there is some redundancy in the measurements- thus permitting statistical analysis of the data.

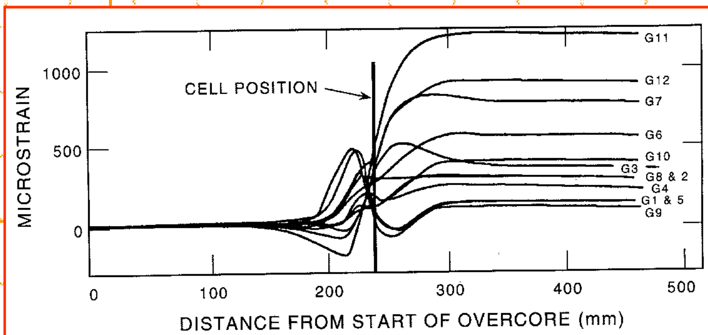
Alternatively, if the rock is assumed to be anisotropic (e.g. transverse isotropic), then the extra readings allow the stress state to be calculated incorporating the rock anisotropy.

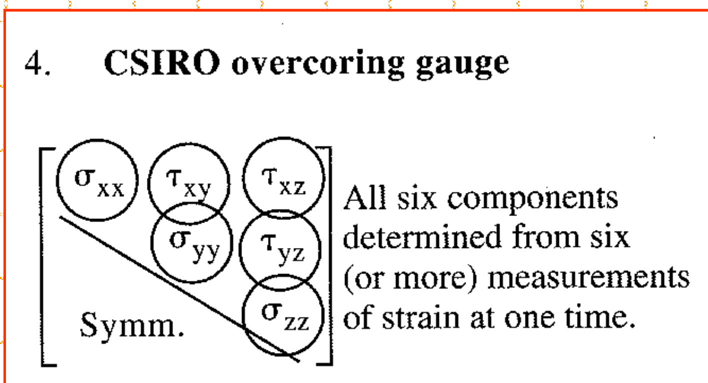


Amadei & Stephansson (1997)

Overcoring Method – CSIRO Hollow Inclusion Cell

The CSIRO measurement cell is one of the few tests that can establish the full stress tensor with one installation.

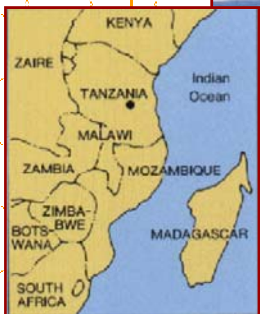


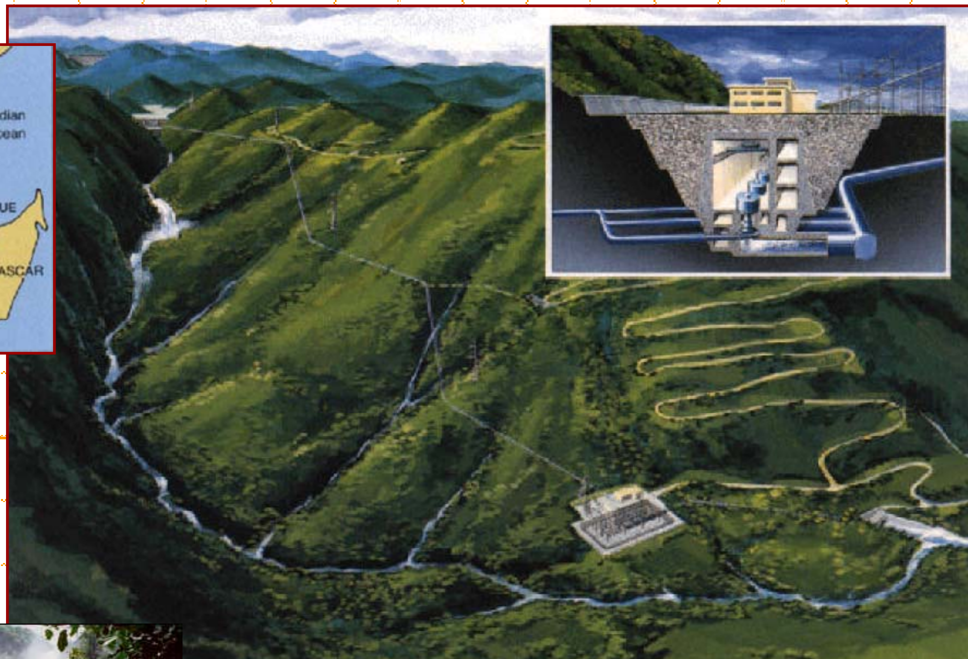


Another advantage of the method is that the hollow rock cylinder can be retrieved and tested under controlled conditions in order to determine the elastic constants and the functionality of the system (e.g. whether strain gauges are properly bonded, whether the test was performed in intact rock, etc.).

One major problem is the environment within the borehole: water or loose material on the borehole walls may hamper bonding of the cell; and drilling fluids may generate temperature effects.

Case History: Lower Kihansi Hydropower Project

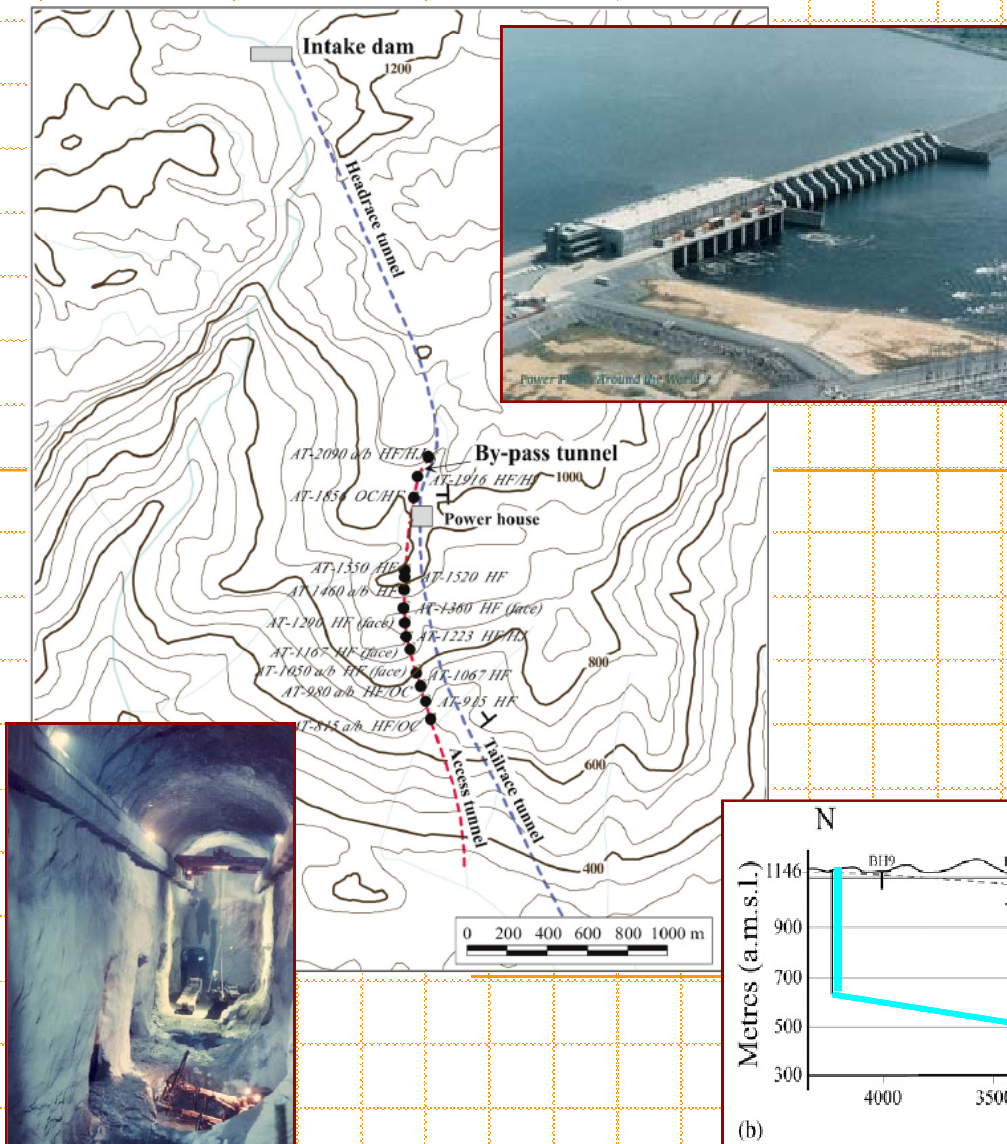




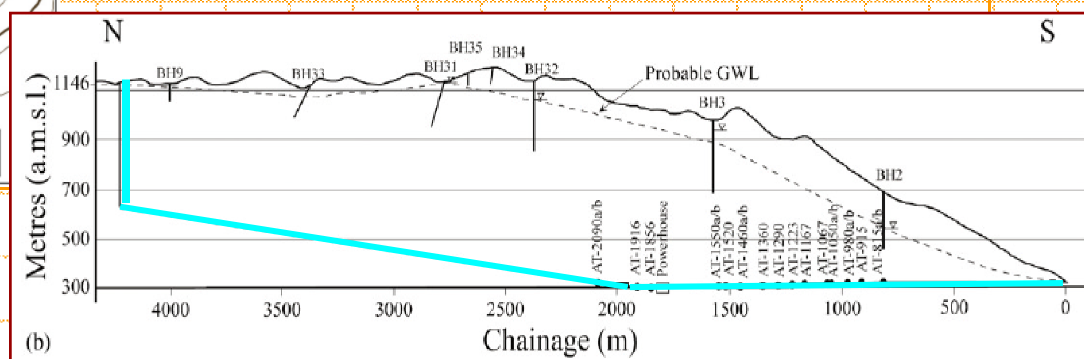
The Lower Kihansi hydroelectric project in Tanzania seeks to utilise the waters of the Kihansi river by channelling part of the river flow upstream of the Kihansi Falls into an inclined high pressure headrace tunnel. The headrace tunnel was planned to be largely unlined.

Unlined tunnels cost 3 to 5 times less than lined tunnels; in this case a cost savings on the order of $10-15 million.

Case History: Lower Kihansi Hydropower Project

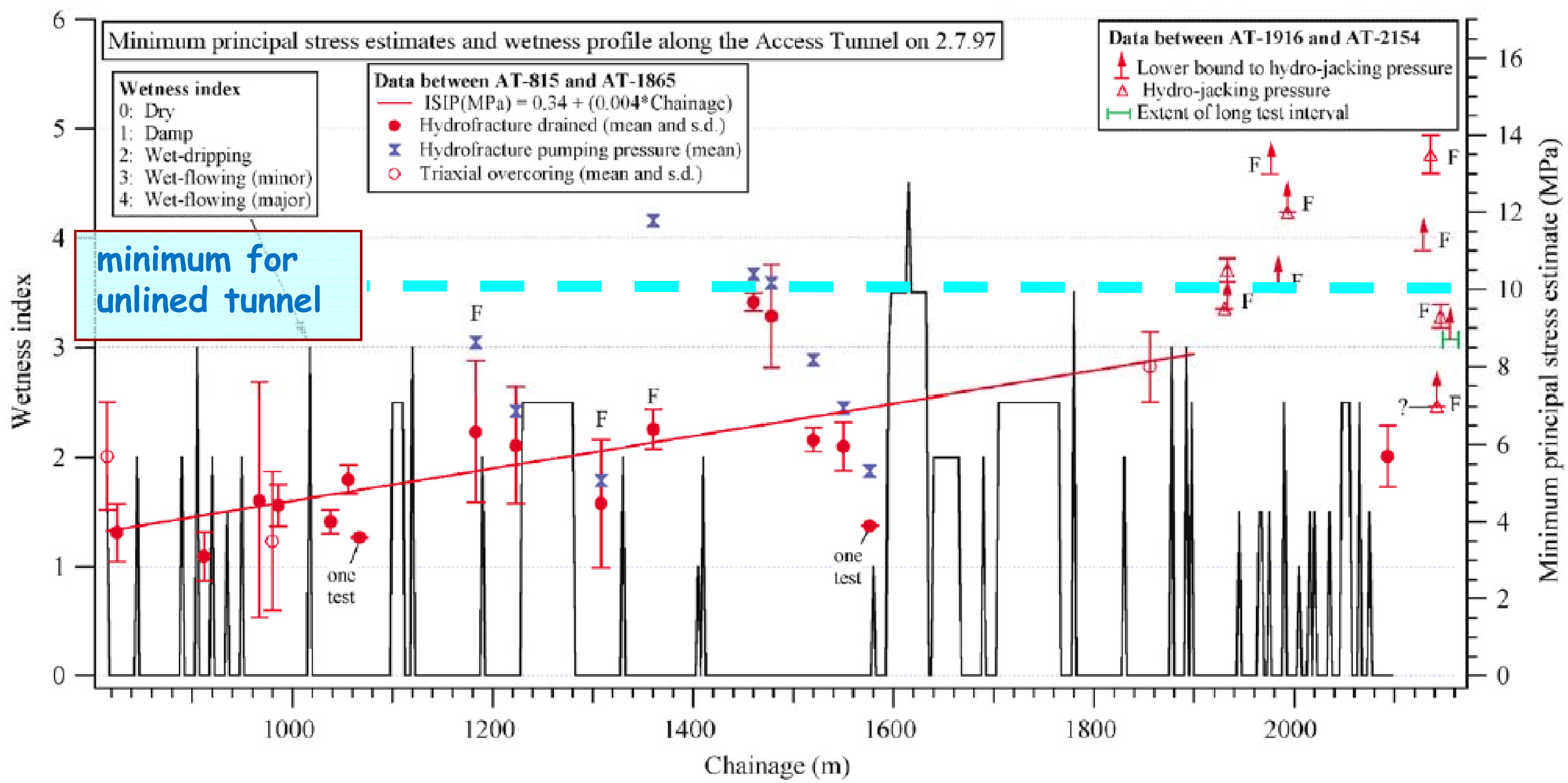


To permit this, the minimum principal stress along the tunnel trajectory under operational conditions would have to be at least equal to the water head times a 1.2 safety factor.



Dahlø *et al.* (2003)

Case History: Lower Kihansi Hydropower Project



Dahlø *et al.* (2003)

Minimum principal stress estimates indicated levels of at least 9.5MPa, increasing to 10.2-11.6 MPa after correction for drainage during testing (thus exceeding the 10 MPa threshold required for an unlined tunnel).

Lecture References

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